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SEVEN LAYERS OF COMPUTATION: METHODOLOGICAL ANALYSIS AND MATHEMATICAL MODELING

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ABSTRACT

We live in an *information society* where the usage, creation, distribution, manipulation, and integration of information is a significant activity. Computations allow us to process information from various sources in various forms and use the derived knowledge in improving efficiency and resilience in our interactions with each other and with our environment. The general theory of information tells us that information to knowledge is as energy is to matter. Energy has the potential to create or modify material structures and information has the potential to create or modify knowledge structures. In this paper, we analyze computations as a vital technological phenomenon of contemporary society which allows us to process and use information. This analysis allows building classifications of computations based on their characteristics and explication of new types of computations. As a result, we extend the existing typologies of computations by delineating novel forms of information representations. While the traditional approach deals only with two dimensions of computation—symbolic and sub-symbolic, here we describe additional dimensions, namely, super-symbolic computation, hybrid computation, fused computation, blended computation, and symbiotic computation.

Keywords: symbol; structure; system; computation; process; symbolic; sub-symbolic; super-symbolic; superstructure; structural machine.

1. INTRODUCTION

The organization of computations in general and the power of operations of the utilized information processing devices have an indispensable impact on the efficiency of computations and the goals that this computation can achieve.

As people are accustomed to computing with symbols, the beginning of information processing technology started with the development of computing devices, which operated with symbols. This trend has been prevailing for quite a while. Therefore, all contemporary computers process symbolic information while digitalization expands to a variety of areas including computers, calculators, tablets, cell phones, TV sets, and servers to mention but a few. Similarly, the development of mathematical models of computation and algorithms started with symbolic systems such as Turing machines or partial recursive functions.

As a result, the symbolic computation was used for modeling the mind and its higher functions intelligence, and cognition, while experts in artificial intelligence formulated the Physical Symbol System Hypothesis (Newell, Simon, 1991), which stated: "A physical symbol system has the necessary and sufficient means for general intelligent action."

However, some researchers, for example, philosopher John Searle, criticized this hypothesis, while the development of information processing theory and technology brought forth another approach to computation and modeling higher brain functions (Searle. 1980). It was connectionism, which was later extended to associationism. According to connectionism, the functioning of the brain is based not on manipulation with symbols but on interactions of the highly connected network of neurons. At the same time, the model of artificial neural networks emerged as an alternative to symbolic information processing. At first, researchers did not regard the functioning of neural networks as computation but later the situation changed and it was assumed that it is subsymbolic or connectionist computation.

Adherents of the connectionist approach to computation in general and to AI in particular maintain that the level of symbolic information processing is too high for many problems and to elaborate an adequate model of the mind and build effective AI systems, it is necessary to utilize subsymbolic computation instead of designing programs that work with symbols.

Here we argue that it is crucial to organize computations not only on two levels—symbolic and subsymbolic but go higher to perform super-symbolic computations and combining all these forms to achieve a superior stage of performance and high level of intelligence. Thus, the goal of this paper is a methodological and philosophical analysis of different pure and combined or aggregated forms of computation.

This paper has the following structure. In Section 2, the concepts of s*ymbol* and *structure* are defined and analyzed. In Section 3, we discuss subsymbolic computations. In Section 4, we reflect on symbolic computations. In Section 5, we determine and explore super-symbolic computations. In Section 6, we describe tools created for operation with structures. Aggregated types of computation are elucidated in Section 7. In Conclusion, we discuss the results of this paper suggesting new directions for research in the theory and practice of algorithms and computation.

2. SYMBOLS AND STRUCTURES

Exploring of the utilization of the term "symbol" in contemporary society, it is possible to find that there are three main interpretations of the word "symbol:"

(1) symbol as a physical object with some meaning,

(2) symbol as a synonym of the concept *sign* being treated as a conceptual structure,

(3) symbol as a conceptual (theoretical) structure and a particular case of signs.

We will call a symbol by the name "*material symbol*" when we have in mind the first interpretation, by the name *conceptual sign* when we bear in mind the second interpretation, and *conceptual symbol* when we take into consideration the third interpretation. In computation, conceptual symbols are represented by material symbols.

Examples of material symbols are printed, written, and displayed on the screen letters, words, digits, and traffic signs.

According to David J. Chalmers, a symbol is an atomic entity, designating some object or concept, which can be manipulated explicitly by a physical symbol system, leading to intelligent behavior (Chalmers, 1992). Symbolic AI deals with the class of programs that perform computations directly upon such symbols.

An example of the case when the terms *symbol* and *sign* are treated as synonyms is the usage of the expressions *symbolic system* and *sign system* although the first one is used much more often.

The third meaning of the word *symbol* as a theoretical or philosophical structure is studied in semiotics as the science of signs. The name *semiotics* comes from ancient Greece where it was assumed that signs exist in nature while symbols function in society. Later Augustine of Hippo (354–430) determined *sign* as a general concept and *symbol* as its particular case in his study of signs and symbols (cf., (Deely, 2009)). An important contribution to this area was the book of John Poinsot (1589–1644) who was also called John of St. Thomas (Poinsot, 1632). The next imperative contribution to semiotics was done by Charles Sanders Peirce (1839–1914) in the form of a general theory of signs (Peirce, 1931–1935; Alp, 2010; Burgin, 2012; 2016; Burgin and Schumann, 2006; Goodman, 1968). Similar to Augustine of Hippo, Peirce treated *sign* as the general term while *symbol* as the convention-based sign constructing the following triadic model of a sign, *Balanced Sign Triad* (cf. Figure 1), where a sign is understood as a relation consisting of three elements: vehicle, object of the sign and meaning.

The Existential Triad of the world (Burgin, 2012) imposes the existence of three *substantial types* of signs and symbols: *material*, *mental,* and *con-*

ceptual signs/symbols, which belong to the physical world, mental word, and the structural world, correspondingly.

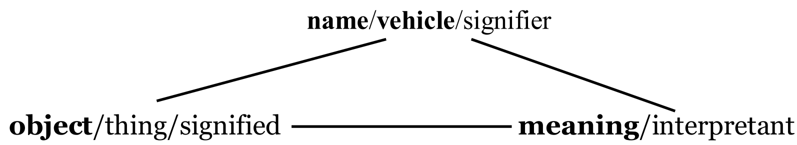

Figure 1. The Balanced Sign Triad or Sign Triangle of Peirce

Usually, when people speak or write about symbols, they mean material symbols. Note that material symbols are not only as individual aggregates of points, such as *a* or 3, written or printed on paper or displayed on the screen. Electrical charges, stones or pebbles can be also material symbols of numbers.

The Balanced Sign Triad of Peirce agreeably correlates with the Existential Triad of the world, which is formed of three basic components: the Physical World, the Mental World, and the World of Structures (Burgin, 2012). In Peirce's triad, the *name* corresponds to the World of Structures as a syntactic system, the *object*/thing corresponds to the Physical World, and the *meaning*/interpretant corresponds to the Mental World as a semantic system. At the same time, the *object* can be non-material and thus, beyond the Physical World. Nevertheless, the object is always closer to the Physical World. On the one hand, this implies that the Balanced Sign Triad of Peirce is homomorphic to the Existential Triad of the world, while on the other hand, it demonstrates fractality of the Existential Triad of the world, which is repeated in a diversity of other natural and artificial systems.

We remind that a *fractal* is a complex system displaying self-similarity across different scales (cf., for example, (Mandelbrot, 1983; Edgar, 2008)). In other words, *fractality* means that the structure of the whole is repeated/reflected in the structure of its parts on many levels (cf., for example, (Coleman and Pietronero, 1992; Calcagni, 2010)). It is possible to find formalized mathematical definition of fractals in (Lapidus, et al, 2017).

According to Peirce, there are three *relational types* of signs: *icon*, *index*, and *symbol*. Thus, as a particular case of signs, a conceptual symbol has the sign triad of Peirce is its structure.

Definition 2.1. An *icon* is an image of the object it signifies.

Photographs at the level of direct resemblance or likeness are prototypical examples of icons. Computer icons helped popularize the word being, as well as the pictographs such as those used on "pedestrian crossing" signs, typical examples of icons. There is no real connection between an object and its icon other than the likeness, so the mind itself is required to see the similarity and associate the two. A characteristic of the icon is that by observing it, we can derive information about its object. The more simplified the image, the less it is possible to learn. No other kind of signs gives that kind of information.

Peirce further divides icons into three kinds:

— *images* have the simplest quality, the similarity of aspect, while portraits, photographs, and computer icons are images.
— *diagrams* represent relationships of parts rather than tangible features, while block schemes, flowcharts, and algebraic formulae are diagrams.
— *metaphors* possess a similarity of character, representing an object by using parallelism in some other object being widely used in poetry and language.

One more type of signs *index* has a causal and/or sequential relationship to its object. A key to understanding indices (or indexes) is the verb "indicate," of which "index" is substantive. For instance, directly perceivable events that can act as a reference to events that are not directly perceivable, or in other words, something visible that indicates something out of sight, are indices. You may not see a fire, but you do see the smoke and that indicates to you that a fire is burning and the smoke is its index. Such words as *this*, *that*, *these*, and *those* are also indices. The nature of the index can be unrelated to that of the signified, but the connection with it is logical and organic, e.g., the two elements are inseparable, and there is little or no participation of the mind to see this connection. Indices are generally non-deliberate, although written or printed arrows are just one example of deliberate ones.

A *symbol* represents something in a completely arbitrary relationship with its object. The connection between the signifier/name and signified/object depends entirely on the observer, or more exactly, what the observer was taught. Symbols are subjective. Their relation to the signified object is dictated either by social and cultural conventions or by habit. Words are the best example of symbols. Whether as a group of sounds or a group of characters, they are only linked to their signified because people decide they are and because the connection is neither physical nor logical, words change the meaning or objects change names as time goes by. Here it all happens in mentality and depends on it.

Note that contrary to Peirce, the French linguist Ferdinand de Saussure (1857–1913) understood the concept *sign* as a subcategory of the concept *symbol*. This relation is represented by the *Dyadic Sign Triad* of Saussure and is presented in Figure 2 (Saussure, 1916).

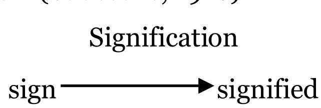

Figure 2. The *Dyadic Sign Triad* of Saussure

Note that this triad is a kind of the fundamental triad (Burgin, 2011).

Indeed, we have the following definition.

Definition 2.2.(a) A *basic named set*, also called a *basic fundamental triad*, is a triad $\mathbf{X} = (X, f, N)$ with the following visual (graphic) representation:

$$X \xrightarrow{\quad f \quad} N$$

(b) A *bidirectional named set*, also called a *bidirectional fundamental triad*, is a triad $\mathbf{X} = (X, f, Z)$ with the following visual (graphic) representation:

$$X \xleftrightarrow{\quad f \quad} N$$

The theory of named sets provides unified foundation of mathematics encompassing set theory, logic, category theory and homotopy type theory as its subtheories (Burgin, 2004). Moreover, it is proved that all mathematical structures, e.g., functions, relations, graphs, categories, functors, operators, and topological spaces, are either named sets or systems of named sets (Burgin, 2011).

Returning to the concept of sign, we see that in contrast to de Saussure and Peirce, Morris defines *sign* in a dynamic way relative to some interpreter. He writes that S is a sign (the sign name) of an object or objects D for an interpreter I to the degree that I takes the account of D in virtue of the presence of S (Morris, 1938). Thus, the object S becomes a sign only if somebody (an interpreter) interprets S as a sign (the sign name). This gives us the following diagram.

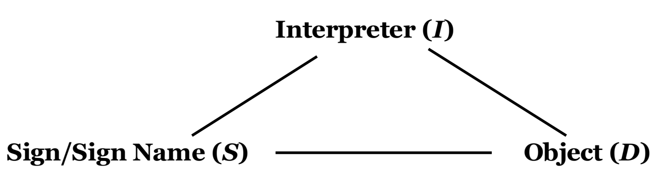

Figure 3. The *Dynamic Sign Triad* of Morris

According to Morris, the sign name is what supports the triadic relation of the sign with other signs, with designated objects and with the subjects using the sign. These relations are represented by the corresponding fields of semiotics.

The Dynamic Sign Triad of Morris also correlates with the Existential Triad of the world. In Morris' triad, the *name* corresponds to the World of Structures as a syntactic system, the *object* can be associated with the Physical World, and the *Interpreter* corresponds to the Mental World as a system mentality that comprehends *S* as a sign (symbol). At the same time, the *object* can be non-material and thus, beyond the Physical World. Nevertheless, the object is always closer to the Physical World. On the one hand, this implies that the Peircean triad is homomorphic to the Existential Triad of the world, while on the other hand, it demonstrates fractality of the Existential Triad of the world, which is repeated in a diversity of other natural and artificial systems.

Observing symbols and signs, we can see their inherent relation to concepts. Indeed, let us look at the Concept Triangle of Russell presented in Figure 4 (Russell, 1905). Then taking Denotation as an Object, we come from the Concept Triangle of Russell to the Balanced Sign Triad (Sign Triangle) of Peirce while interpreting Object as Denotation, we come from the Sign Triangle of Peirce to the Concept Triangle of Russell.

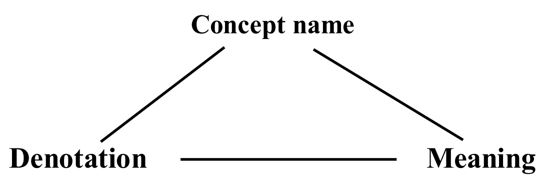

Figure 4. The *Concept Triangle* of Russell

The most advanced model of concepts—the Representational Triad—presented in Figure 5 is homomorphic to the Dyadic Sign Triad of Saussure (Burgin and Gorsky, 1991; Burgin, 2012). At the same time, both of them are particular cases of the fundamental triad.

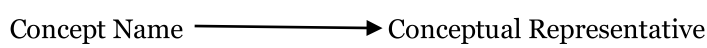

Figure 5. The Representational Triad of a Concept

Signs, symbols, and concepts are structures. Computations are performed by operation with structures. That is why we present the exact definition of a structure.

Traditionally it is defined as follows (cf., for example, (Robinson, 1963; Grossmann, 1990; Tegmark, 2008)):

Definition 2.3. A *structure* is (a representation of) a complex entity that consists of parts in relations to each other.

However, it was demonstrated that this definition is essentially incomplete. A more comprehensive formal definition was suggested by Bourbaki in the case of mathematical structures (Bourbaki, 1957).

It is necessary to remark that Bourbaki (1957; 1960) also elaborated a formal definition of a general mathematical structure as a very abstract concept. Here we provide a short description of the formal definition from (Bourbaki, 1960) omitting some formal details and expressions.

Bourbaki start their definition of a structure Σ with a finite sequence of ordered pairs of whole numbers, calling this sequence an *echelon construction scheme*. Then taking such a scheme S and n terms $E_1, E_2, E_3, \dots, E_n$, which denote (name) sets, in a formal theory **T** that is stronger that a theory of sets, such as ZF, they build an *echelon construction* E of the scheme S on the sets $E_1, E_2, E_3, \dots, E_n$, which are taken as the building blocks of the inductive construction employed by Bourbaki. Each step of this construction consists either of taking the Cartesian product ($E \times F$) of two sets obtained in the preceding steps or of taking the power set 2^D of the set D obtained in the previous steps. This echelon construction of the scheme S is a sequence of terms in the theory **T** built according to the scheme S. After building this construction, Bourbaki take (in the theory **T**) a formal representation of a group of mappings f_i: $E_i \to E_i'$ ($i = 1, 2, 3, \dots, m$) and determine *canonical extensions* with the scheme S of the mappings f_i to a mapping of an echelon construction E of the scheme S on the sets $E_1, E_2, E_3, \dots, E_n$.

After this, Bourbaki characterize a *typification* T of letters $x_1, x_2, x_3, \dots, x_n$ in **T**. Subsequently, they delineate the concept of a *transportable relation* with respect to T. Next Bourbaki define (1) a *species of the structure* Σ in **T** as a text that is a combination of letters $x_1, x_2, x_3, \dots, x_n, s$, terms in **T**, (2) a typification $T\{x_1, x_2, x_3, \dots, x_n, s\}$ of the letters $x_1, x_2, x_3, \dots, x_n, s$ in **T** called the *typical characterization* of the species of the structure Σ, and (3) a relation $R\{x_1, x_2, x_3, \dots, x_n, s\}$ that is transportable with respect to T and called the *axiom* of the species of the structure Σ.

To define a "species of structure" Σ, Bourbaki take:

(1) n sets $E_1, E_2, \dots, E_n$, as "principal base sets."

(2) m sets $A_1, A_2, \dots, A_m$, the "auxiliary base sets", and finally

(3) a specific echelon construction scheme $S(X_1, X_2, \dots, X_n, A_1, A_2, \dots, A_m)$.

All these auxiliary constructions and definitions allow Bourbaki to define the *structure of species* Σ, taking terms $E_1, E_2, E_3, \dots, E_n$ in the theory **T** as principal base sets. Namely, this construction leads to the following definition (Bourbaki, 1960).

Definition 2.4. A term U in the theory **T** is called a *structure of species* Σ if the relation

$$T\{E_1, E_2, E_3, \dots, E_n, U\} \& R\{E_1, E_2, E_3, \dots, E_n, U\}$$

is a theorem in **T**.

Covering several pages in the book (Bourbaki, 1960), the completely formalized formal definition of a structure in the sense of Bourbaki is essentially much more complex and much longer than the partially formal definition given above. Besides, this definition is too abstract and complicated even for the majority of mathematicians, who prefer to use an informal notion of a mathematical structure or the definition where a structure is formalized as a set with relations in this set. As Corry (1996) writes, Bourbaki's concept of *structure* was, from a mathematical point of view, a superfluous undertaking. Even Bourbaki themselves did not use this formalized concept in their later books of the *Eléments* after they had introduced it in *Theory of Sets* (Bourbaki, 1960). However, being overcomplicated, this definition is still incomplete. For instance, this definition does not discern inner and outer structures.

The complete formal definition of a structure was developed in the general theory of structures including both formal and informal forms (Burgin, 2012). Here we give only an informal definition of set-theoretical structures. There are also mereological structures, which are defined and explored in (Burgin, 2012).

Definition 2.5. A *structure R* consists of elements/parts and connections/relations of the three categories:

— Connections/relations between (groups of) elements/parts
— Connections/relations between (groups of) elements/parts and (groups of) connections/relations
— Connections/relations between (groups of) connections/relations

Note that elements themselves can be and often are structures. This property is called nesting and is used in many processes to improve their efficiency (Burgin, 2020).

Structures, elements of which are also structures, are called *superstructures*.

According to the general theory of structures (Burgin, 2012), there are three *existential types* of structures:

1. Ideal structures
2. Abstract structures
3. Embedded structures

Embedded structures are structures of physical systems (things), such as tables, trees or cars, and of mental systems, such as thoughts or values. An interesting example of a structure embedded in the mentality of society is a moral space studied in (Boltuc, 2013).

Abstract structures exist in the mentality of people or groups of people and are characterized only by their properties.

Ideal structures dwell in the world of structures described for example in (Burgin, 2017).

This world is the scientific incarnation of the world of Plato Ideas (Forms). Indeed, for millennia, the enigma of the world of Ideas or Forms, which Plato suggested and advocated, has been challenging the most prominent thinkers of the humankind. The solution to this problem was found only recently. Namely, an Idea/Form in the Plato's sense can be interpreted as a scientific object called a structure. The difference is that Plato Ideas/Forms do not have a rigorous inclusive definition while structures have an accurate definition in the general theory of structures (Burgin, 2012). Based on this definition, it was demonstrated that structures have the basic properties of Plato's Ideas. In addition, it was proved the existence the world of ideal structures (Burgin, 2017).

It is also important that it was possible to discover the most basic atomic structure in the world of structures. It is called *fundamental triad* or *named set* (Burgin, 2011). Its definition is given in Section 2. Any structure is either a fundamental triad (named set) or is built of some number of fundamental triads (named sets). It means that the discovery of fundamental triad (named set) actually accomplished the search at first of philosophers and later of physicists for the entity out of which everything in the world is built. In this sense, the theory of named sets is the theory of everything (Burgin, 2011).

Assessing the place of structures in the world and their roles, it was found in the general theory of structures (Burgin, 2012) that systems have five substantive types of structures: *inner*, *internal*, *intermediary*, *outer,* and *external* structures.

According to the general theory of structures, we have the following definitions of these types:

Definition 2.6. (a) An *internal structure TQ* of a system *R* contains only inner structural parts, components and elements, i.e., parts, components and elements of *R*, relations between these parts, components and elements, relations between these parts, components, elements and relations from *TQ* and relations between relations from *TQ*.

(b) An *inner structure IQ* of a system *R* is a substructure of an internal structure *TQ* of *R*, where *IQ* is obtained by exclusion of (1) the whole system *R* as a part, component or element of itself and (2) all relations that include *R*.

(c) An *external structure EQ* of a system *R* is an extension of the internal structure, in which other systems, their parts, components and elements are included, as well as relations between all these included parts, components and elements, relations between these parts, components, elements and relations from *EQ* and relations between relations from *EQ*.

(d) An *intermediate structure MQ* of a system *R* is a substructure of an external structure *EQ* of *R*, where *MQ* is obtained by exclusion of (1) the whole system *R* and other systems from *EQ*, as well as (2) all relations that include these systems.

(e) An *outer structure OQ* of a system *R* is an inner structure of a system *U* in which *R* is only one of the inner elements of the inner structure *IQ* of the system *U*.

It is also possible to classify structures by their elements. It results in three pure classes of structures:

— *Subsymbolic structures* have only elements that are not treated as symbols but as parts of symbols
— *Symbolic structures* have only elements that are treated as symbols
— *Super-symbolic structures* have elements that are assembled from symbols

Examples of subsymbolic structures are sets of pixels on the screen of a computer, tablet or TV set.

Symbolic structures are composed of symbols in a simple way, that is, these structures have low structural complexity. Symbols, words, texts as a linear composition of words, and sets are symbolic structures.

Letters and digits are paradigmatic examples of symbols while words and numerals are examples of symbolic structures.

Texts, hypertexts, and diagrams are examples of super-symbolic structures.

In addition, there are *mixed structures*, which have elements of both types:

— *Hybrid structures* have both elements that are comprehended as symbols and elements that are treated as parts of symbols
— *Fused structures* have both elements that are recognized as symbols and elements that are operated as super-symbolic structures, for example, as assemblages of symbols
— *Blended structures* have both elements that are identified with subsymbols and elements that form super-symbolic structures, for example, as assemblages of symbols
— *Symbiotic structures* have elements of all three pure types

According to the general theory of structures, there is a hierarchy of structures composed of different orders of structures (Burgin, 2017).

Let us consider the mathematical formalization of the two first levels of this hierarchy.

Definition 2.7 (Burgin, 2012)**.** A *first-order structure* is a triad of the form

$$\boldsymbol{A} = (A, r, \boldsymbol{R})$$

In this expression, we have:

— the set A, which is called the *substance* of the structure $\boldsymbol{A}$ and consists of elements of the structure $\boldsymbol{A}$, which are called *structure elements* of the structure $\boldsymbol{A}$

— the set $\boldsymbol{R}$, which is called the *arrangement* of the structure $\boldsymbol{A}$ and consists of relations between elements from A in the structure $\boldsymbol{A}$, which have the first order and are called *structure relations* of the structure $\boldsymbol{A}$

— the *incidence relation* r, which connects groups of elements from A with the names of relations from $\boldsymbol{R}$

Examples of structures of the first order:

The order relation: 1 < 2 < 3 < 4 < 5
A string: $a - b - c - d - e$
A word: $s - e - v - e - n$

Definition 2.8 (Burgin, 2012)**.** A *second-order structure* is a triad of the form

$$\boldsymbol{A} = (A, r, \boldsymbol{R})$$

Here

— the set A, which is called the *substance* of the structure $\boldsymbol{A}$ and consists of elements of the structure $\boldsymbol{A}$, which are called *structure elements* of the structure $\boldsymbol{A}$

— the set $\boldsymbol{R}$, which is called the *arrangement* of the structure $\boldsymbol{A}$ and consists of relations in the structure $\boldsymbol{A}$, which are called *structure relations* of the structure $\boldsymbol{A}$

— r is the incidence relation that connects groups of elements from A and/or relations from $\boldsymbol{R}$ with names of relations from $\boldsymbol{R}$

— $\boldsymbol{R} = \boldsymbol{R}_1 \cup \boldsymbol{R}_2 \cup \boldsymbol{R}_3$

— $\boldsymbol{R}_1$ is the set of relations between the elements from the set A

— $\boldsymbol{R}_2$ is the set of relations in the set $\boldsymbol{R}_1$, i.e., elements from $\boldsymbol{R}_2$ are relations between relations from $\boldsymbol{R}_1$

— $\boldsymbol{R}_3$ is the set of relations between elements from A and relations from $\boldsymbol{R}_1$

Relations from $\boldsymbol{R}_2$ and $\boldsymbol{R}_3$ are called relations of the second order in $\boldsymbol{A}$.

Second-order structures are used to represent data processed by structural machines of the second order.

Similarly, we determine relations and structures of higher orders.

Examples:

1. The strict order < on numbers is a suborder of the non-strict order ≤ on numbers. This is a relation of the second order.

2. Addition and subtraction are ternary relations. Subtraction is inverse to addition. This is a relation of the second order.

3. A function is a binary relation. When one function is an extension of another function, it defines a relation of the second order.

4. 0 is neutral element with respect to addition. This is a relation of the second order.

5. Taking relations between people, when the relations between A and B are better than the relations between A and D, it defines a relation of the second order.

In information technology, supercomputers are computers that have essentially better characteristics of information processing in comparison with ordinary computers. Usually, the improved characteristic is the higher speed of computing. In a similar way, superstructures are structures that have essentially higher complexity.

Symbolic superstructures are composed from symbols and symbolic structures. Intricate hypertexts, operational schemas, multicomponent images, and structures of higher order are symbolic superstructures.

Now let us analyze how operating with different types of structures shape specific types of information processing in general and computation in particular.

3. SUBSYMBOLIC COMPUTATIONS IN NATURE AND ARTIFICIAL DEVICES

Although there are different definitions of subsymbolic computation, here we uphold the following definition.

Definition 3.1. *Subsymbolic computation* is computation in which elements of processed data are not interpreted as symbols or sets of symbols by the computing system.

This well correlates with the opinion that in a system performing subsymbolic computation, the objects of computation are more fine-grained than the objects of semantic interpretation (Chalmers, 1992).

For instance, it is often assumed that the tokens manipulated by neural networks performing primitive operations are subsymbolic as they are located at a level lower than that of the symbols (Rumelhart, McClelland, 1986; Smolensky, 1988). Examples of such tokens are the activation values of neurons. In many cases, in a system performing subsymbolic computation, the computational level lies beneath the representational level (Chalmers, 1992).

Starting from the last quarter of the 19th century, there was an assumption in computer science, artificial intelligence and cognitive sciences that neural networks, which represent the connectionist model, perform subsymbolic computations. For instance, Smolensky introduced the following Subsymbolic Hypothesis as "the cornerstone of the subsymbolic paradigm":

> "The intuitive processor is a subconceptual connectionist dynamical system that does not admit a complete, formal, and precise conceptual level description." (Smolensky, 1988)

A paradigmatic example of subsymbolic computations is provided by artificial neural network, which lately became extremely popular due to their ability to perform deep learning.

Analog computing is another important form of subsymbolic computations.

In comparison with symbolic computation, subsymbolic computation has the following advantages (Kwasny, Faisal, 1992):

— It is more robust in noisy conditions
— Provides better performance for analog data
— It demands less knowledge upfront
— It is easier for scaling up
— It better adapts to Big Data
— It is better for perceptual problems
— It is more useful for building models in neuroscience

Indeed, now the prevailing opinion of neuroscientists is that intuitive mental processes that internal functioning of the brain does not utilize a symbolic description but require subsymbolic descriptions inherent for connectionist architecture. As a result, the subsymbolic paradigm provides better means for modeling the capabilities of the brain, which also implies reduction of mental to neural computation.

4. SYMBOLIC COMPUTATIONS AS THE BASIC FORM OF ALGORITHMIC INFORMATION PROCESSING

Now we come to symbolic computations. In contrast to subsymbolic computation, in a system performing subsymbolic computation, the objects of computation are also objects of semantic interpretation and very often the computational level coincides with the representational level (Chalmers, 1992).

Definition 4.1. *Symbolic computation* is computation in which elements of processed data are interpreted as symbols by the computing system and computation is performed by the individual transformation of these symbols.

Turing machines are the paragon of automata performing symbolic computations. Indeed, on each step of its computation a Turing machine, observes a symbol in a cell of its tape and eventually changes this symbol to another symbol before moving to another cell.

In comparison with subsymbolic computation, symbolic computation has the following advantages (Kwasny, Faisal, 1992):

— In it, introspection more useful for coding
— It is easier to debug
— It is easier to understand and explain
— It is easier to control
— It is more efficient for solving abstract problems
— It is better adapted for explaining people's thinking

At the same time, the symbolic/subsymbolic distinction does not imply the architectural dissimilarity (Chalmers, 2018). Indeed, on the one hand, Turing machines can and contemporary digital computers do simulate neural networks, which are paradigm examples of subsymbolic computation. On the other hand, neural-network can precisely model Turing machines (Siegelman, 1999).

Connectionist paradigm also does not contradict the possibility to imply it using symbolic computation. Indeed, cellular automata have the connectionist architecture but each cell of these automata performs symbolic computation (cf., for example, (Burgin, 2005)).

Now we can describe the new type of computation.

5. SUPER-SYMBOLIC COMPUTATIONS AS A NEW DIMENSION OF INFORMATION PROCESSING

Analysis of real-life computations show that computers and other advanced information processing systems, such as the brain, operate not only with symbolic and subsymbolic data but also with essentially more advanced structures.

Definition 5.1. Computation is *super-symbolic* when symbolic structures of higher orders and superstructures are transformed as holistic objects.

This contrasts symbolic computations where symbolic structures are transformed by operating with separate symbols.

Super-symbolic (transcendent) computation is a model of functioning of the right hemisphere of the brain. Indeed, processing images of material systems by transformations of holistic shapes is an example of super-symbolic computation.

One more important example of super-symbolic computation is operation with schemas (Burgin, Mikkilineni, 2021). These processes are very important for the functioning of the mind because the framework of schema theory can provide a better bridge from human psychology to brain theory than that offered by the symbol systems (Arbib, 2021).

The advantage of the super-symbolic (transcendent) computing is its ability to operate big formal and informal systems of data and knowledge with high efficiency. That is why the implementation of super-symbolic computing is the way to the solution of the problem of big data and information overflow.

6. AMALGAMATED LEVELS OF COMPUTATION

The combinations of pure types give mixed types of information processing. The first step in this direction gives us *hybrid computation*, which comprises both symbolic and subsymbolic computations being a two-fold type of computations (Burgin, Dodig-Crnkovic, 2015). Hybrid computation allows combining advantages of both symbolic and subsymbolic computations.

Researchers found that individual neurons can perform symbolic computations (Cepelewicz, 2020; Gidon, et al, 2020). For instance, it was discovered that individual dendritic compartments can also perform a particular computation—"exclusive OR"—that mathematical theorists had previously categorized as unsolvable by single-neuron systems.

Moreover, some psychologists assume that the roots of arithmetic reside in single neurons (Dehaene, 2002). It means that neural networks in the brain perform both symbolic and subsymbolic computations, i.e., they operate is on the level of hybrid computation.

Conventional models of computation perform either symbolic computation, e.g., finite automata, Turing machines, inductive Turing machines or Random Access Machines (RAM), or subsymbolic computation, e.g., neural networks or cellular automata. New models, such as neural Turing machines (Graves, et al, 2014; Collier, Beel, 2018) or structural machines with symbolic and subsymbolic processors, carry out hybrid computation.

A neural Turing machine is a recurrent neural network with a network controller connected to external memory resources. As a result, it combines subsymbolic computation of neural networks with symbolic computation of Turing machines.

Super-symbolic (intuitive) computation adds one more dimension to the general schema of computational processes. This allows merging this type with already known types brings us to the system of *three twofold types* of computation:

— *hybrid computation* combines symbolic and subsymbolic computation

— *blended computation* combines subsymbolic and super-symbolic computation

— *fused computation* combines symbolic and super-symbolic computation

While it is easy to understand how information processing systems, such as computers or the brain, can perform fused computations, realization of blended computation looks more intriguing. One way to do this is simply to utilize two types of processors in the computing system—processors of one type work with subsymbolic data whereas processors of the other type operate super-symbolic data. Another mode of blended computation has two stages. At the first stage, the computing system processes subsymbolic input data developing one or several super-structures. These super-structures are handled at the second stage of the computational process. For instance, a neural network can aggregate or find an operational schema and then this schema is used and transformed, for example, improved, by an appropriate assembly of neural networks.

Synthesizing super-symbolic computation with symbolic (rational) computation and subsymbolic (intuitive) computation in one model, we come to *symbiotic computation*. Structural machines with flexible types of processors can accomplish symbiotic computation. Symbiotic computation allows combining advantages of all three pure types of computation representing the entire type of computations.

Thus, there is also one entire type of information processing:

— *symbiotic computation* combines all three pure types of information processing.

It is possible to consider symbiotic computation as the highest level of computation as it comprises all other types of computation.

7. OPERATING WITH STRUCTURES AND SCHEMAS

The identification of the new types of computation needs machines that would be able to perform such computations. Structural machines provide means for all types of computation including symbiotic computation when the machines possess processors of different types (Burgin, Adamatzky, 2017; Burgin, 2020). Let us describe these powerful models of computation.

A structural machine M works with structures of a given type and has three components:

1. The *control device* C_M regulates the state of the machine M
2. The (entire) *processor* P_M performs transformation of the processed structures and its actions (operations) depend on the state of the machine M and the state of the processed structures. The entire processor consists of one or several unit processors. When a structural machine is considered only as a theoretical model, it is possible that the entire processor contains infinitely many unit processors.
3. The *functional space* Sp_M consists of three components:
   - The *input space* In_M, which contains the input structure(s).
   - The *output space* Out_M, which contains the output structure(s).
   - The *processing space* PS_M, in which the input structure(s) is transformed into the output structure(s), which form the results of computation of a structural machine.

Unit processors can move in the processing space performing operations with structures in their neighborhoods according to the rules of their structural machine. Unit processors can function in the centralized mode when they are regulated by the common centralized control device. When the structural machine has the distributed control device, which consists of several unit control devices, the unit processors of this machine can function in two modes: clusterized and totally distributed modes. In the clusterized mode, all unit processors of the structural machine are divided into several groups (clusters) and each group works with its own control device. In a totally distributed mode, each unit processor has its individual control device. This architecture of the structural machine allows considerable flexibility and adaptivity.

Unit processors of one structural machine can be of different types and categories. For instance, it is possible that one unit processor is a Turing machine, another unit processor is a neural network, while the third one is a cellular automaton and one more unit processor is an inductive Turing machine.

It is natural to assume that all structures—the input structures, the output structures and all processed structures—have the same type.

The computation of a structural machine M determines the *trajectory of computation*, which is a tree in general case and a sequence when the computation is deterministic and is performed by a single processor unit.

8. CONCLUSION

Information, and the computing structures that process it, play a critical role in how we, as humans, perceive the structural reality that surrounds us and how we interact with it. The Existential Triad of the world derived from the general theory of information describes the three worlds that interact with each other (Burgin, 2012). First, we have the material world, where structures exist and obey the laws of conversion of energy and matter. Biological systems have through evolution, and natural selection developed information processing structures that receive information about other material structures through various senses they have developed using their physical structures.

In addition, there is a mental world encompassing mental structures. Mental structures allow living systems to create and use their "vital potentialities and life processes." According to the general theory of information, knowledge derived from information can be represented in the form of ideal structures consisting of an atomic structure called the "fundamental triad." The fundamental triad (also known as a name set) consists of entities, relationships, and behaviors caused by actions and events that change the state of the entities performing information processing in general and computation in particular.

In this paper, we have discussed two types of computational structures namely, symbolic and subsymbolic computation, which are ubiquitous in the current state of the art in information technologies. Symbolic computing deals with the evolution of structures made up of symbols and subsymbolic computing deals with elements of processed data that are not interpreted as symbols or sets of symbols by the computing system. Deep learning using a neural network model is an example.

In addition, we have also analyzed super-symbolic computation where structures as holistic objects are processed in contrast to symbolic computation where sequences of symbols are processed. This approach has many advantages (Burgin, Mikkilineni 2021) going beyond current symbolic and subsymbolic computational methods used in information technologies. The highest type—symbiotic computation allows us to use symbolic, subsymbolic, and super-symbolic computations. An application of symbiotic computing using symbolic, subsymbolic, and super-symbolic computing is discussed in Burgin-Mikkilineni Thesis (Burgin, Mikkilineni, 2021; Mikkilineni, 2022). The other three forms of computing are fused computation, blended computation, and hybrid computation, which are also observed in nature.

In this paper, we have presented a new and comprehensive picture of information structures, information processes (computations), and the associated tools derived from the general theory of information. We hope that this will guide us to not only understand how we as humans process and use information, but also will allow us to build a new class of digital automata that mimic how people process information.

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