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THE COMPUTATIONAL AND PRAGMATIC APPROACH TO THE DYNAMICS OF SCIENCE

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ABSTRACT

Science means here mathematics and those empirical disciplines which avail themselves of mathematical models. The *pragmatic* approach is conceived in Karl R. Popper's *The Logic of Scientific Discovery* (p. 276) sense: a logical appraisal of the success of a theory amounts to the appraisal of its *corroboration*. This kind of appraisal is exemplified in section 6 by a case study—on how Isaac Newton justified his theory of gravitation. The *computational* approach in problem-solving processes consists in considering them in terms of computability: either as being performed according to a model of computation in a narrower sense, e.g., the Turing machine, or in a wider perspective—of machines associated with a non-mechanical device called "*oracle*" by Alan Turing (1939). Oracle can be interpreted as computer-theoretic representation of *intuition* or *invention*. Computational approach in another sense means considering problem-solving processes in terms of *logical gates*, supposed to be a physical basis for solving problems with a reasoning.

Pragmatic rationalism about science, seen at the background of classical rationalism (Descartes, Gottfried Leibniz etc.), claims that any scientific idea, either in empirical theories or in mathematics, should be checked through applications to problem-solving processes. Both the versions claim the existence of abstract objects, available to intellectual intuition. The difference concerns the dynamics of science: (i) the classical rationalism regards science as a stationary system that does not need improvements after having reached an optimal state, while (ii) the pragmatical version conceives science as evolving dynamically due to fertile interactions between creative intuitions, or inventions, with mechanical procedures.

The dynamics of science is featured with various models, like Derek J. de Solla *Price's* exponential and Thomas Kuhn's paradigm model (the most familiar instances). This essay suggests considering Turing's idea of oracle as a complementary model to explain most adequately, in terms of exceptional inventiveness, the dynamics of mathematics and mathematizable empirical sciences.

Keywords: algorithm, behavioral (vs declarative) knowledge, computability, corroboration, innate knowledge, intuition, invention, logic gates, oracle, pragmatic (vs classical) rationalism, problem-solving, reasoning, symbolic logic, Turing machine.

1. WHAT DOES IT MEAN "COMPUTATIONAL" AND WHAT "PRAGMATIC"?

1.1.

This essay is meant to sketch some fundamentals of the *computational worldview*—as one being best suited to the realities of the era of computerization.[1] Such a modern worldview gets realized in a possibly best way by what I call *computational and pragmatic rationalism*. This is the contention of the present essay.

Let us start from realizing that we happen to live in a new civilizational period—the era of computerization. Each era, in spite of diversity of opinions, ideologies, programs, etc. is featured with a characteristic *Zeitgeist*, and it finds expression in some dominating worldview. This can be said about the Middle Ages, Renaissance, Enlightenment, the industrial era, and so on. Such a *Zeitgeist* depends from the current state of knowledge, social and economic conditions, common opinions and endeavours.

At all the listed points the time of ours has brought far-reaching changes, even revolutionary, which require an effort to form a worldview for new times, ready to answer its unprecedented challenges. The choice of the term *computational* worldview is motivated by the obvious reason that "computation" and "computability" are key concepts in the era of computerization.

However, there a single and unique system of assertions to deserve this naming does not exist. The name "computational worldview" encompasses a fairly diversified class of views. They have in common a concept of computation, but may differ considerably with regard to the preferred model of computation, relations between different models, and so on.

In that class of computing-oriented worldviews there is one which deserves to be called *modern rationalism*. An inspiring sketch of topical rationalism is found by Kurt Gödel. Well suited for developing Gödel's approach is the model of computation devised by Alan Turing (1939). It was meant by Turing as a sequel to the computing model known as the Universal Turing Machine (for short UTM), defined in the fundamental study (1936). It is this enhancement what I am to to discuss here, jointly with Gödel's ideas.

Such an attempt to synthesize Gödel's and Turing's contributions—as far as I know—is rather innovatory, not likely to be found in the current literature. As being fairly new, this project may be debatable. Such a debate should check if the suggested here synthesis is well-founded.

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[1] See (Stacewicz 2016). Stacewicz's term "informational worldview" is equivalent with the adopted in this paper "computational worldview." Numerous proposals and comments on this issue take place at the academic forum *CAFE ALEPH – Polemics and Chats about Computational Worldview*. The set-theoretical term "Aleph" alludes to Turing's (1936) proof concerning uncountable numbers which deals with a relation between Aleph-zero and Aleph-one. See also (Stacewicz 2019).

To explain the role of the word "modern," we need to consider it at the historical background of classical rationalism, typically represented by Plato and the great thinkers of the 17th century, mainly Descartes and Gottfried Leibniz. The both branches have in common the claim about the existence of abstract objects (sets, numbers, etc.), available to intellectual intuition. However, in the modern approach the concept of intuition is split into two distinct abilities. One of them I am to call "inventive," and the other "innate" or "inborn."

The inventive intuition does not appear in any system of classical rationalism, while the innate is the subject of intense reflection on the knowledge *a priori* assumed to be an inborn ability. Reflections concerning the existence and infallibility of "a priori" form an important chapter in the doctrine of traditional rationalism.

On the other hand, according to the modern rationalism the inventive intuition plays a key role in the progress of science—conceived as the increase of power to solve ever more problems and ever harder ones. In this context, solvable means capable of being solved with mechanical (algorithmic) computation. Here we see again a deep reason to apply the term "computational" to the modern rationalism.

The distinction between two varieties of intuition, the attributing of distinct functions to each of them, and connecting this differentiation with two historical types of rationalism, is yet another novelty contributed with this essay, and one that invites a critical discussion.

1.2.

A next innovation that may seem controversial consists in noticing a relationship between these two features attributed to modern rationalism. To wit: being computational and being pragmatic.

The latter term refers to the epistemological doctrine of pragmatism as stated, mainly, by Charles Sanders Peirce and Henry James. *Pragmatism* decidedly opposes the claim characteristic of classical rationalism: that there are judgments beyond any doubt, endowed with the virtue of absolute infallibility. Such are—according to that claim—evident principle given a priori to the mind, and those which are entailed by such principles. As stressed by the 17th century rationalists, such are axioms of mathematical theories and their consequences. This view, an integral part of classical rationalism, is called *infallibilism*.

Obviously, the view denying infallibilism merits the name of *fallibilism***.** It was Peirce who has laid the foundations of fallibilism when observed that that there was no need of infallible principles of a general nature, addressed to the whole mankind. For, it in the nature of any inquiry, that is, of a process of problem solving, that as a researcher I have no chance, no possibility, to start from premises not being my own. Thus, even if there were principles regarded by other people as infallible, but not convincing for myself, I would be entirely unable to take of them any advantage or assistance. Thus there is no need of any universal infallibilist creed. What is necessary to properly fix one's own belief, is by Peirce expressed in his seminal article where he states the following:

> "It is a very common idea that a demonstration must rest on some ultimate and absolutely indubitable propositions. These, according to one school, are first principles of a general nature; according to another, are first sensations. But, in point of fact, an inquiry to have that completely satisfactory result called demonstration, has only to start from with propositions perfectly free from any actual doubt. If the premises are not in fact doubted at all, they cannot be more satisfactory than they are" (Peirce 1877, 6).

This is a typically pragmatic argument from impracticability, that is, the impossibility of doing something effectively. Peirce addresses every researcher with the following message: do not bother whether the premises used in your demonstration belong to some infallible principles. Even if they were so, but were doubted by you personally, it would be practically absurd to make any use of them, when you possess certainties of your own, entirely sufficient to be used as premises.

In confrontation with indisputable empirical facts, or with new well-founded achievements in mathematics, such personal certainties have to recede, and be replaced by credible new data. Then the researcher, if get convinced about a new reliable result, does revise his premises to gain new chances of cognitive success. Again, in such a strategy, no need arises to resort to some principles supposed to be infallible.

1.3.

Infallibilism happens to be associated with the epistemological analytic-synthetic dichotomy, where infallible propositions are at the analytic side, and fallible on the synthetic. Unfortunately, the conceptual situation is entangled for the unclear status of analytic propositions, e.g., their relation to a priori judgments, especially in the face of Kant's conception of synthetic a priori statements. These, according to Kant, are factual (synthetic) and, at the same time, universally and necessarily true, hence infallible, as exemplified with the basic propositions of geometry. The issue of real definitions still contributes to the puzzle, since as real they ought to be synthetic, while as definitions—analytic. As a special case of real definitions may be seen axioms in their role of characterizing the senses of primitive concepts of a theory.

Anyway, when being aware of such complications, we can bypass them with assumption that in the standard (i.e., non-pragmatist) epistemologies some propositions are regarded as infallible, and they are dichotomously separated from those being fallible. Only the existence of such a dichotomy is what matters in the present discussion, independently of how the elements of dichotomy are conceived.

As for pragmatism, it denies the dichotomy of fallible and infallible items of knowledge, the former represented by empirical sciences, the latter—by logic and mathematics. Instead of the dichotomy, the rationalistic pragmatism, championed by Kurt Gödel and Willard Van Orman Quine (1953), suggests to see the set of scientific theories as ordered according to the degrees of fallibility. Thus the least fallibility amounts to the *highest reliability* being attainable, but not necessarily requires an entire lack of fallibility.

For example, let us compare as to the degree of reliability: set theory, number theory and logic. Set theory is below the two remaining, e.g., for its problem with antinomies, and for involving such controversial items as the axiom of choice and the continuum hypothesis.

Giuseppe Peano's arithmetic advantage over set theory can be explained in a typically pragmatist way—its successful countless applications in the practice since thousands of years, without ever committing a smallest error. However, among various PA variants there appears a gradation. For instance, some mathematicians have a greater trust in the reliability of the first-order axiom of induction, than in the cases its higher orders.

Such a gradation has not only a due theoretical justification, but also a methodological advantage, over dichotomy. Even if the scale of degrees of reliability does not yield a linear order, then a partial order will do to make reasonable choices between propositions or whole theories. If, for instance, a nominalist refuses to accept the second-order axiom of induction, then he is free to rely on the first-order version as supposed by him—according to his philosophical belief—to have a higher degree of reliability.

In the presence of such a ladder of reliability degrees, there arises the question: which discipline or theory may enjoy the highest attainable reliability? This is to mean: which of them is the closest to the top, namely, to the ultimate (i.e. absolute) reliability which amounts to infallibility? The candidate most likely to win is the first-order classical logic of predicates. Such highest precedence is to mean that in the case of contradiction between a logical law and any other statement, it is the latter which should be rejected as false.

How such a dominance could be explained? What about attributing logic the feature of being innate? If it proves to be innate, then how is this feature related to being *a priori*? These issues are to be discussed in the next section.

2. HAS SYMBOLIC LOGIC THE HIGHEST RELIABILITY DUE TO IMPLEMENTATION OF LOGIC GATES IN BRAIN CIRCUITS?

2.1.

There was mentioned (1.1) the importance of intuition which is featured by *creative inventiveness in solving problems*, but not always privileged with high reliability, that is, not unlikely to fail is a process of problem solving. Now it is in order to focus on another kind: not inventive intuition but having, instead, the advantage of highest reliability.

The argument to be offered in what follows is to the effect that there exists a kind of intuition even if not infallible, then closest to infallibility. However not so much spiritual (as Plato or Descartes believed) but rooted in some inborn structures in animal brains. And that logic, even as sophisticated as modern symbolic logic is rooted in those biological traits that we acquired, first as part of our primate heritage, and further developed as we evolved.

An important route of evolution leads from instinctive *protological behavior* up to the hights of symbolic logic and computer science. I take advantage of the term "protological," as defined in *Lexico UK Dictionary*: *relating to an early, basic, or undeveloped form of logic.*

This handy concept is what we need in the present discussion. It makes possible to consider degrees of logical competence, and use this term to denote its lowest degrees in which no verbal expression and even no awareness is involved. Then we can trace the evolutionary chain of links which leads from the lowest to the highest level.

When observing a problem-solving behavior of an animal in their search for food, fights with rivals, escaping dangers, etc., we perceive the strategy which looks as guided by logical rules, in particular: generalization, instantiation, detachment, transposition.

What a conclusion should be drawn by a logician making such observations? Suppose, he is aware that his own strategies in the problem-solving behavior would be like those adopted by animals. Both in the cases of instinctive or automated thoughtless problem-solving, as in the cases of solving the same problem thoughtfully and with awareness of inference rules, the logician finds the same logical rules.

If the problem, for instance, to find a method to reach an object which in the moment is beyond the reach is given to a chimp, to a logician, and to a computer, their process of problem-solving reveals the same logical schema. A report concerning reasonings of non-human animals is given by Marciszewski (1994/2012, chap. 7) who comments on the famous Köhler's research in the intelligent behavior of a chimpanzee named Sultan.

In order to solve the problem of reaching a fruit being in the moment beyond the actual reach, Sultan behaves is such a way as if knew the logical rules: of forming conjunction, of instantiation and of detachment. In the mentioned book the author attempts to simulate Sultan's reasoning with the use of computer. The intention was to check the correctness of this reasoning (with the checker called Mizar-MSE), and to learn whether there is a parallelism between the supposed pieces of behavioral logic and of symbolic logic implemented in the computer.

2.2.

The experiment of simulating on computer chimp's problem-solving suggests a conjecture to explain the likeness of logical schemas in the reasonings of humans, animals and machines. The familiar von Neumann's architecture contains logic gates implementing Boolean functions, and those provide symbolic logic with firm algebraic foundations. Owing to them, computers can compute and reason in an infallible way.

Independently of von Neumann's computer architecture, where logic gates are basic element of computing and reasoning machines, analogous structures were detected in the central nervous system by the logician Walter Pitts and the neurologist Warren McCulloch. This surprising result was published in the article bearing the much speaking title: *A Logical Calculus of Ideas Immanent in Nervous Activity* (1943). In computers logic gates are connected with wires, in brains they are nerve cells connected with axons.

This result has revealed that the operations of reasoning and calculating are ruled by the laws of Boolean algebra, on an equal footing in humans and in machines. The logic based on this algebra provides means to formalize any computing or reasoning, and once something can be formalized it can be mechanized, either with electronic or with biological machines.

An intriguing question with which philosophers would wish to address biologists and cognitive scientists, is concerned with computing and reasonings performed by non-human animals: are their brains equipped with logic gates too?

If the answer were in the affirmative (as it seems to be in some research reports), then philosophers would be ready to claim that logic is omnipresent in the live nature. On this basis the pragmatist argument could be coined that the nature provided a lot of its creatures with logic as excellent means to fight for survival. Their successful applications in that fight would convincingly confirm the validity of Boolean rules of reasoning, independently of their intellectual evidence. Having had so countless empirical confirmations, logic could pride oneself on winning much more scores than other science in the endeavour to possibly highest reliability.

Such a key role of logic gates in the animal world could be also used as a case for nativism—a significant constituent of rationalistic philosophies. The competence owed to logic gates, inherited after parents and a chain of ancestors, would testify the innateness of logical skill.

The above remarks are a kind of thought experiment to account for the existence of logical knowledge in the naturalistic vein as recommended, e.g., by Jan Woleński (2016). He tries to bridge logic and cognitive science from a naturalistic point of view, to oppose classical rationalism—criticized firstly by Peirce.

When conjecturing the existence of logic gates in the brains of non-human animals, I do this with the intention of checking its mettle. Let us do our best to support this bias toward naturalistic epistemology, and look to what degree is it feasible. In such an inquiry it will be in order to confront the naturalistic approach to nativism with its opposite, represented by classical rationalism in several versions, each rooted in a different metaphysical vision.

There is Plato's answer taking advantage of the legend of the soul's preexistence and remembering (anamnesis) the knowledge attained in that phase. There is Augustinus' claim in terms of divine illumination. And that of Descartes who instead of divine illumination speaks of *lumen natural*—the light somehow endowed by Nature to human minds. Descartes extensively and systematically featured the role of what he called *intuitus*.

As for Leibniz, his Monadology presents monads conceived as preprogrammed living entities—*divina automata*, or *divinae machinae* (his own words) with a suitably equipped memory. It is Leibniz who merits attention as a forerunner of *nativism* worth to be remembered in the era of computerization. There is a thought-provoking analogy between his point and the definition of the adjective *preprogrammed* in current dictionaries. This definition, when referred to living creatures, runs as follows: *preprogrammed = genetically biased towards a particular behaviour*. Thus nativism, when associated with the idea of automaton or machine, manifests itself as a likely component of *computational* rationalism.

If there is a bias toward a conditioned genetically behaviour, then it belongs to the innate traits. Combining genetics and the theory of automata, somehow on the Leibnizian pattern, Chomski revolutionized the current linguistics and philosophy of language, pioneering thereby the modern rationalism. His concept of *linguistic competence* denotes *the ideal language system that enables speakers to produce and understand an infinite number of sentences* in their language, and to distinguish grammatical sentences from ungrammatical sentences. The infinity should be understood here as an countably infinite set.

2.3.

Analogously to Chomski's concept of linguistic competence there appeared in cognitive psychology, and philosophy of mind, the notion of *logical competence* to name cognitive mechanism that enables to complete logical tasks. For instance, Paula Quinon writes in her paper *Logical Compe-*

tence: “Systems of core cognition correspond to what is called competences. [...] Systems of core cognition are present in infants and also shared with many non-human animals. This means they are pretty deeply inserted in the brain structure (Quinon, no date of publication).”

The phrase “logical competence” is convenient for an analogy with Chomski’s notion of linguistic competence, and agrees with defining it in dictionaries as ability to do something efficiently. The stress put on biological foundations of logical competence and their innativeness (as inborn to infants and animals) does seem justified in the light of current knowledge about brain structures.

However, there is a significant disparity between linguistic and logical competences. The former is a feature of human minds alone, while the latter—as remarked in the above comment—is possessed also by non-human animals.

Some people doubt whether non-human animals, even so intelligent as cats, dogs, chimps etc. are capable of having a logical competence. The doubt may arise when no distinction is made between *behavioral knowledge* (often called procedural) and *declarative knowledge*. In a more idiomatic form, popularized by Gilbert Ryle (1949), the counterparts of these technical terms are, respectively, “knowing how” and “knowing that.”

The distinction is nicely mirrored in the domain of logic with regard to humans. Every human being avails himself with *behavioral logic* while professional logicians and their audiences know additionally its declarative counterpart such as symbolic logic. Though the latter is beyond any reach of non-human animals, are there really any reasons to refuse them behavioral logic? Even everyday observations, as well as professional experimental inquiries, hint at the animal abilities of solving problems in such a way as if they knew inference rules of detachment, transposition, instantiation, etc. (compare the story of Sultan told in 2.1).

Human beings share such a behavioral logic with chimps as if a kind of protological anticipation. At the same time, however, they are privileged with an enormous advantage—that of being language-speaking creatures. Owing to that, they could make an astonishing evolutionary leap—to transform their behavioral logic into declarative logic, symbolic and formal, and thus enter a decisive route of civilizational development.

The first system of formal logic, that of Aristotle, in its long historical development has led to Boolean algebra which contributed to the rise of Gottlob Frege’s axiomatic system of logic. That, in turn, together with Bertrand Russell’s and Giuseppe Peano’s achievements, led to David Hilbert’s program. It stimulated the astonishing Gödel’s and Turing’s discoveries, paving the way to the theory of computability and the rise of computer science.

Such an immense civilizational epic wouldn't happen if the mankind did not inherit after its animal ancestors the innate behavioral logic which was to become the source and truth warrant for declarative formal logic. The maximal reliability of such a warrant stems from the fact that behavioral logic has found an unimaginable number of confirmations, having been so successfully applied by animals in their fight for survival during the millions years of evolution. This is the best possible pragmatic check of the highest reliability of logical intuitions displayed in an unconscious logical behaviour of animals.

3. HILBERT'S PROBLEM OF THE DECIDABILITY OF LOGIC, TURING'S FORMAL MODEL OF THE DYNAMICS OF SCIENCE

3.1.

The most consequential problem about mathematical intuition put Hilbert (1928) under the name *Entscheidungsproblem*. Immediately it is concerned with the power of algorithms apt to be expressed in predicate logic, but indirectly it has far-reaching consequences for the concept of intuition. Before discussing the issue more extensively, it will be in order to sketch the core of argument.

This is the problem of algorithmic decidability of formalized predicate logic: does there exist a mechanical procedure to decide about any of its formulas whether it is a logical tautology or is not. When in their 1928 textbook Hilbert and Ackermann stated the question, such procedures were already invented for propositional calculus, but not for predicate logic. The authors emphasized that the problem is of fundamental significance, and seemed to expect its positive solution in a not distant future.

The solution appeared after few years, due to several authors who independently in the same year 1936 published their results. The most seminal were the results of Turing whose the basic part appeared in (1936) and the sequel in (1939). The former has brought the most influential model of computing known as the Universal Turing Machine, mentioned in 1.1. As commonly known it is in no need to be here reported.

The study of 1939 (also announced in 1.1) will be now discussed from the angle of its relevance to the issue of scientific dynamics which entered a dramatically new phase through Turing's discovery of algorithmic undecidability of symbolic logic and analyzing its epistemological consequences in the study on ordinal logics. Its full title reads: *Systems of Logic Based on Ordinals* (1939), and the main idea is the following:

> The well-known theorem of Gödel shows that every system of logic is in a certain sense incomplete, but at the same time it indicates means whereby from a system L of logic a more complete system L' may be obtained. By repeating the process we get a sequence L, L_1 = L', L_2 = L'_1, L_3 = L'_2,... of logics each more complete than the preceding.

To grasp this main idea, one should focus on considering an infinite *ordered* sequence of logical systems ever stronger, that is, having ever greater problem-solving ability. To get more to the heart of the matter, the above fundamental statement should be read in the light of the following passage in section 4:

> "Let us suppose that we are supplied with some unspecified means of solving number theoretic problems; a kind of *oracle* as it were. *We will not go any further into the nature of this oracle than to say that it cannot be a machine;* with the help of this oracle we could form a new kind of machine (call them *o*-machines), having as one of its fundamental processes that of solving a given number theoretic problems."

Through this suggestive picture of oracle Turing introduces the revolutionary idea of *relative computability* to highlight the busy road of the progress of mathematics (that supports significantly the progress of the rest of knowledge). Since in formalized systems, including Turing machines, computing is the universal method of problem solving, the relativeness of computability entails relativeness of solvability.

Before there appeared these surprising results, no scientist imagined such a gradation of solvability. Optimists like Hilbert believed in the maxim *in der Mathematik gibt es kein "ignorabimus,"* while those less optimistic divided the set of problem into disjoint and closed classes: solvable and non-solvable.

Thus, for long time the potentiality of such dynamic migration of unsolvable problems to the class of solvable was weakly felt by a majority of scholars. The growth of such awareness can be observed among computer scientists, as suggestively expressed Salomon Feferman's (1992) article entitled: *Turing ˙s Oracle: From Absolute to Relative Computability and Back*. More details about the impact of the idea oracle Feferman gives (2006). Martin Davis (2006) states that Turing's use of a computing oracle has proven to be highly influential in theoretical computer science, e.g., in the polynomial time hierarchy.

While mathematicians and computer scientists more and more appreciate the idea of oracle, as Feferman reports in the cited article (2016), philosophers try to interpret this new mathematical idiom in terms of epistemology, epistemology, psychology, philosophy of mathematics.

How far have we progressed in these domains owing to the notion of oracle? Does such a vision represent a realistic model of dynamics of science? May there exist, in the real world, physical or mental entities to form such an infinite sequence of ever more potent problem-solvers?

In the literature dominates interpretation to the effect that oracle is an idealized model of mathematical intuition. This approach is shared by such experts as Roger Penrose and Andrew Hodges (see (Copeland 2008) referred to in footnote 2).

There is an impressive evidence given by Max H. Newman who in a biographical memoir on Turing, identifies the oracle with mathematical intuition. Newman was Turing's collaborator, and had to know his intentions—to the effect that the oracle enables to see the truth of a formally unprovable Gödel statement; in this task human intuition does succeed.[2]

Turing (1939) also connected achievements of mathematical intuition with the progression of ordinal logics, when making the following comment:

> "Owing to the impossibility of finding a formal logic which will wholly eliminate the necessity of using intuition we naturally turn to 'non-constructive' systems of logic which not at all the steps are mechanical, some being intuitive. An example of a non-constructive logic is afforded any ordinal logic. When we have an ordinal logic we are in a position to prove number theoretic theorems by the intuitive steps."

3.2.

It would be welcome to get acquainted with concrete instances of the envisioned by Turing progression of ever stronger problem-solvers. From this point of view, there seem to be relevant Gödel's consideration of the infinite ordered sequence of logics of ever higher orders. The higher is the order of a system, the greater its deductive power—exactly in the sense defined by Turing: a system marked by a natural number, say n denoting the order of a logic, is able to solve every problem solvable by those bearing a number lesser than n, and additionally some problems that cannot by solved by any of its antecessors.

Moreover, such a new system has the very desirable merit not having been mentioned by Turing. The increase of the deductive power results in a significant shortening of problem-solving procedures. Here is Gödel's own statement. It has been demonstrated, not by Gödel himself, but by other authors some years later. It was S. R. Buss (1994) who produced a detailed

[2] See: https://www.ics.uci.edu/ welling/teaching/271fall09/Copeland—TheMathematicalObjection.pdf—Jack Copeland (2008) created a very useful and interesting collection of statements on intuition and creativity made by Turing, Gödel, Post and Penrose, entitled *The Mathematical Objection: Turing, Gödel, and Penrose on the Mind*.

proof, while George Boolos offered a nice exemplification in his seminal study *A Curious Inference* (1987). A philosophical comment on Boolos' inference is found in (Marciszewski 2006), while its computer implementation of this inference is due to Benzmüller and Brown (2007).

The latter research demonstrates enormous advantages of higher-order logic (or a corresponding system of set theory) with regard to the length of proof. They powerfully demonstrate what Gödel (1936) says about shortening of proofs "by an extraordinary amount" in the following statement.

> Thus, passing to the logic of the next higher order has the effect, not only of making provable certain propositions that were not provable before, but also of making it possible to shorten, by an extraordinary amount, infinitely many of the proofs already available.

As to the power of the higher-order logic, a striking exemplification can be found in case of arithmetic. Consider arithmetic formulated in the language of second-order logic. The belonging to that order makes it possible to quantify not only over natural numbers (as individuals) but also over sets of natural numbers. Since real numbers can be represented as infinite sets of natural numbers, and since second-order arithmetic allows quantification over such sets, the theory of real numbers can be formalized in second-order arithmetic; see (Sieg 2013, 291). Such a close assimilation to the theory as powerful as mathematical analysis is a remarkable achievement of the second-order logic.

How is related the Gödel's claim to the idea of oracle? Certainly, the formalized systems of logic of ever higher orders can be regarded as machines, since formalization, practically, equals mechanization, as suggestively expressed by Gregory Chaitin (2006): "Gödel's 1931 work on incompleteness, Turing's 1936 work on uncomputability, and my own work on the role of information, randomness and complexity have shown increasingly emphatically that the role that Hilbert envisioned for formalism in mathematics is best served by computer programming languages, which are in fact formalisms that can be mechanically interpreted (Chaitin 2006)."

The above account is an attempt to exemplify the general Turing's schema with concrete cases. How far such concretizations are relevant to the issue of mathematical intuition as a source of potent algorithms, it is the issue open to a further penetrative discussion.

4. MODERN RATIONALISM AS AN OPTIMISTIC, AS WELL AS REALISTIC, VISION OF THE DYNAMICS OF SCIENCE

4.1.

Does science progress? Those who have grown in the cultural environment of the 20th and the present century may be surprised that somebody puts such a question. It seems as pointless, as if somebody asked, for instance, whether a triangle should contain three angles. For we conceive the scientific progress as belonging to the very nature of science like the triangularity to the essence of a triangle.

A different concept of science was characteristic, in particular, of the Middle Ages. Then the whole work of scholars was devoted to the preservation, transmission and commenting the body of knowledge inherited from antiquity. The long process toward our current awareness was due to many intertwined factors.

The one especially relevant to the present discussion it is the growing demand for new, ever more efficient and more numerous methods of calculation. This pression, typical for civilizational development, was coming from astronomy, engineering, navigation, economy, etc. In the 20th century it culminated inside mathematical logic in the idea of great reform of mathematics. How there has come to this brainchild, is a story to be told in this discussion.

Why logic played a major role? The road from a prescientific, solely intuitive, mathematical theory, as practiced, say, in the ancient Egypt, Babylon, etc. up to its doing in a mechanized way, as in our era of computers, leads through two preparatory phases: axiomatization, and then formalization in a language in which we could express the whole of mathematics.

Such a language, envisioned by Leibniz, has been accomplished first by Frege (1879), and then by Russell, Peano, Hilbert, Gerhard Gentzen, and Polish logicians. This is the language of predicate logic, capable of expressing—with the help of suitable definitions and substitutions for variables—every mathematical proposition. Half century after Frege, owing to the genius of Turing (1936), we have got the message that the predicate logic is capable of being implemented in a machine to prove theorems and compute mathematical functions.

However, Turing's achievement which fulfilled the hope in the possibility of mechanizing calculations and reasonings, at the same time brought entirely unexpected result about serious limitations of computing machines. Using Cantor's diagonal method, Turing proved the existence of uncomputable functions. When asked about the value of such a function, the machine does not bring any result, and cannot halt the procedure, making infinite loops.

This amounts to the undecidability of predicate logic–the issue mentioned above (2.2) in connection with the problem of ascertaining logical truths. Every process of computing the value of a function can be interpreted as the proof of an arithmetical theorem. If the value of a function cannot be computed, this means that the corresponding theorem cannot be proved. The existence of unprovable theorems amounts to undecidability of logic.

4.2.

However, Gödel believed in human reason's ability to make a concept more and more precise, up to the point in which it can be characterized by axioms of a theory. The fact of its being formalized ensures an algorithmic procedure to solve problems which were unsolvable before axiomatization and formalization. Such a process can be nicely exemplified by the history of making the concept of set ever clearer—from a vague idea up to the stage of formalization, e.g., with Zermelo-Fraenkel set theory.

Tu sum up, Hilbert's claim that [H] *there is an algorithm to computationally solve every mathematical problem*, has been replaced by Gödel's claim that, owing to intellectual intuition, [G] *for each mathematical problem there can be found an algorithm to solve it*. This difference becomes more conspicuous when expressed with logical formulas; in the following, the variable *a* runs over the set of algorithms, while *p* – set of problems.

[H] $\exists_a \forall_p (aSp)$

Computational Non-Pragmatic Rationalism – CNPR

[G] $\forall_p \exists_a (aSp)$

Computational Pragmatic Rationalism – CPR

Formula G expresses just a part of CPR. It hints at the difference with H which consists in the order of quantifiers. For the full characterization of CPR, it should be added that the algorithm *a* to solve problem *p* is not always at hand (as is in the case of CNPR), but has to be found in a process which starts from an act of intuition. Thus the existential quantifier in G means something like potential existence. That such a potentiality is real, is an optimistic feature of CPR. Gödel advocated CPR as an "optimistic rationalism" (his own phrase). This optimistic attitude is penetratively analyzed by Stacewicz (2019, sec. 5).

Such an optimism involves the conjecture about the reliability of mathematical intuitions. However, there are philosophers and even circles of philosophers, as the Vienna Circle, that do not admit any trace of intuition as a factor in what they call "scientific philosophy." They reject intuition as misleading and needless. Nowadays such opinions remain influential not so much among mathematicians, as among some representatives of humanities.

4.3.

The inquiry into the said issue leads to acknowledging the indispensability of mathematical intuition on par footing with the indispensability of algorithms, in the drive of science toward ever higher solvability. This drive is admirably efficient, as we see in the history of science and in our everyday lives. And its efficiency speeds up every year, in particular, in natural sciences and technology. Why there is so?

The first step towards the answer is to realize that nowadays both, science and technology, enjoy a solid and extensive mathematical basis, one that didn't exist, neither was thinkable in any earlier time. Owing to such an excellence, it can bring ever more numerous and more surprising results. Let me hint as the two astonishing and spectacular cases.

Among them there was in 2015 the detection of gravitational waves which round hundreds years earlier were predicted by Einstein on the purely mathematical ground as equations of general relativity, but up to the year 2015 conjecture fact could not be approached experimentally for the lack of suitably sensitive instruments.

In turn, Einstein's theory would not arise in 1915, if there did not exist a perfectly suited for such a purpose non-Euclidean elliptic geometry, created in 1866 by Riemann for pure theoretical reasons, without any inkling about revolutionary empirical applications to come in a future.

The case is exceptionally intriguing for its nexus with the issues of geometrical intuition. Let us consider the following remark in *WolframMathWorld*:[3]

> "In three dimensions, there are three classes of constant curvature geometries. All are based on the first four of Euclid's postulates, but each uses its own version of the parallel postulate. The 'flat' geometry of everyday intuition is called Euclidean geometry (or parabolic geometry), and the non-Euclidean geometries are called hyperbolic geometry (or Lobachevsky-Bolyai-Gauss geometry) and elliptic geometry (or Riemannian geometry). Spherical geometry is a non-Euclidean two-dimensional geometry. It was not until 1868 that Beltrami proved that non-Euclidean geometries were as logically consistent as Euclidean geometry."

Without any polemical comment, I would just encourage those who dislike talking of intuitions that them try to replace the phrase "everyday intuition" by any other, being more "scientific" according to their standards of scientific exactness.

Anyway, let us take for granted the existence of everyday geometrical intuition, akin (presumably) to some rudiments of procedural (imperative) knowledge possessed by other mammals. Higher animals seem to enjoy a similar orientation in space, though they did not study Euclid. It looks as if were an inborn rudiments of geometry in animals.

This is not to mean that non-Euclidean geometries contradict the everyday geometrical intuition—presumed in Euclid's work. At the bottom of

[3] See: http://mathworld.wolfram.com/Non-EuclideanGeometry.html

non-Euclidean approaches there are other intuitions, connected with astronomical observations, as exemplified with the case of Ptolemy. He was aware that the area of a triangular region on the sphere is precisely the amount by which its angle sum exceeds 180 degrees.[4]

Thus geometrical intuitions stemming from an astronomical experience differ from those of everyday experience, but there does not occur between them any contradiction (see the passage of *MathWorld* cited above). Anyway, to start a cognitive process, we have to rely on some primordial insights.

4.4.

Having had devoted a bit of attention to geometrical intuition, it is in order to mention the issue of intuition in arithmetic—as much as needed to highlight a pragmatic approach to the problem.

As we can observe in primitive tribes and in children, arithmetic starts from perceiving small sets of physical things. A child perceives differences of set size (number of elements) in some cases and size identity (equinumerosity) in other ones. The latter is necessary to form the notion of (cardinal) number. The other factor, not less necessary is person's capability of abstracting.

This capability should be acknowledged as an inborn ability, indispensable for learning a native language through the procedure of ostensive definitions. The role of abstraction in the procedure of ostensive defining is too extensive theme to be considered in this essay. A fairly exhaustive treatment is found in the book by Marciszewski (1994, chap. 8) in the chapter entitled "The ostensive procedure as a paradigm of definition."

After gaining the notion of natural number, people are able to imagine the successor of any number, and successor of that successor, and so on, potentially up to the infinity. It is a remarkable and even mysterious feature of humans, one that made them able to climb higher and higher the ladder of mathematical abstraction.

Quite different is an approach to arithmetic which has been popular because of having a famous author—Immanuel Kant. He regarded arithmetic as the knowledge based on the *pure intuition* of time. This way of thinking is presently continued in the philosophy of mathematics termed intuitionism. However, this doctrine does not seem to accord with what we know about cultural evolution of mankind: the process which starts from sensory observations of small sets, not having yet the concept of zero. Then due to a long evolution the awareness of humans (at least some of them) reaches the heights of set theory, and logic with arithmetic logic of arbitrarily high order, and so on.

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[4] See: http://www.math.brown.edu/ banchoff/Beyond3d/chapter9/section03.html

These impressive achievements are not confined to pure theory. As a rule, such intellectual insights lead to a well-confirmed empirical theories by devising a calculus suitable for the domain in question, in order to compute functions which render empirical laws—natural, social, mental, etc. Such was the case of Isaac Newton, Albert Einstein, Werner Heisenberg, Erwin Schrödinger, von Neumann etc. For instance, to establish an algorithm of rational decision making (for economics, praxiology, etc.), we need the calculus of probability, while physics resorts to geometrical models, differential calculus, etc.

Let us sum up the role of intuition in the algorithm-oriented progress of science with Chaitin's suggestive statement to run as follows:

> "There is no limit to what mathematicians can achieve by using their intuition and creativity instead of depending only on rules of logic. Any important mathematical question could eventually be settled, if necessary by adding new fundamental principles to math, that is, new axioms or postulates. This implies that the concept of mathematical truth becomes something *dynamic* that evolves, as opposed to the traditional view that mathematical truth is static and eternal" (Chaitin 2006).

5. EXTENSIONAL VS INTENSIONAL EQUIVALENCE OF MODELS OF COMPUTATION FROM THE ANGLE OF SCIENCE DYNAMICS

This distinction is to the point in the debate about the strong AI project and its influence on the understanding of the dynamics of science. It allows to briefly express the strong AI claim by saying that the human brain is extensionally equivalent to the Universal Turing Machine, without being equivalent intensionally.

As for alternative computation models, as cellular automata, artificial neural nets, analog computers, etc., they—according to the Strong AI doctrine—should be reducible to UTM. Reducing means here: to regard those alternatives as extensionally equivalent with UTM (as the paradigmatic case).

To briefly explain the distinction, I avail myself with its concise formulation by Paweł Stacewicz who sums up a more detailed text by Hajo Greif *Invention, Intension and the Extension of the Computational Analogy* posted on "Cafe Aleph"—an academic forum to discuss philosophy of computer science:[5]

> "Two models of computation are extensionally equivalent if they have the same class of solvable problems (regardless of how these problems are

[5] Both texts are available when addressed: http://marciszewski.eu/?p=10558

solved). Thus: the Universal Turing Machine (UTM) model is equivalent to both the recursive functions model and the quantum computation model. In contrast, the UTM model is not extensionally equivalent to the analog-continuous model of computation (described by means of real recursive functions). The latter, theoretically speaking, allows to solve the TM halting problem (unsolvable under the UTM model). It is therefore extensionally stronger" (Stacewicz, see footnote 5).

To extend the list of main extensional equivalences with UTM, let us complete it with Church's lambda-calculus and Post's systems. The same cases are also examples of *intensional non-equivalence*—the term needed to account for the fact that some models, though extensionally equivalent obtain the same results in a different way. The Strong AI doctrine holds that the human brain is extensionally equivalent to UTM, but admits that it may be not equivalent intensionally.

Let us employ the phrase "*scientific robot*" to name any Turing machine programmed to do science. According to the Strong AI doctrine, such robots can be produced when the complexity of electronic agents will match that of human brains. Some Strong AI adherents, for instance Ray Kurzweil, hold that the ability to produce such agents should appear soon, near 2050.

Suppose that after 2050 the task of doing science should be performed by scientific robots. Thus their producers have to solve the problem of equipping them with the trait of inventiveness. This would be a crucial issue for predicting the future dynamics of science. If the project does succeed, then the dynamics of science will be like that having been hitherto. If it happens to fail, then the Strong AI project proves utopian, and the task of dynamically forwarding science would remain with humans, since there is no progress of knowledge without creative invention.

The distinction we here discuss, though useful in comparing models of computation, demands a more precise explication. It turned out so, for instance, when there occurred a problem with publishing Turing's (1936) study. The editor was not sure whether the study was duly original, or it repeated—only with a different terminology—Alonzo Church's result stated in terms of lambda-calculus.

This meant the doubt whether their results are equivalent not only extensionally (what later proved evident) but intensionally as well; were the latter the case, this would mean lack of originality. Only after Turing submitted additionally a proof of intensional non-equivalence, the study on computational numbers could accepted for publication.

Let it be added that the very term "extensional"—whose understanding is needed to grasp the meaning of "extensional equivalence of computation models"—is pretty familiar to logicians. The historical origin of this concept goes back to Frege and his famous comparison of the phrases "morning star" and "evening star"—equivalent extensionally and different intensionally. Persons less familiar with the issue, may consult relevant reference works.[6]

The opposition discussed above provides the opportunity to render concisely this essay's main point that is as follows. If there existed extensional equivalence between UTM and the mind/brain as a model of computation, and the science would be done by scientific robots, then the dynamics of science would disappear. This would be unavoidable for the lack of curiosity, imagination and inventiveness as being the privilege of humans alone. This point is developed in the two next sections.

6. WOULD THERE BE ANY DYNAMICS OF SCIENCE, IF THE SCIENCE WERE BEING DONE BY MACHINES? NEWTON'S GRAVITATION AS A CASE STUDY

6.1.

The title of this essay promises considering the progressive dynamics of science which more and more furthers its frontiers. In the preceding sections only the dynamical evolution of mathematics was handled, hence now it is in order to pay attention to empirical sciences.

The former so extensive treatment of mathematics is dictated by the fact that it was metamathematics in which one worked out the conceptual apparatus to deal with progress in terms of the efficiency of problem-solving methods. In turn, this speedup of *solvability* was explained with reference to the logico-mathematical notion of *computability*. The latter does not belong to the standard vocabulary of the methodology of empirical science where solvability is addressed with some related concepts: induction, probability, confirmation, corroboration, etc.

Nevertheless, the issue of computability is firmly present in the deep structure of empirical theories. Mainly in physics, but also in some social sciences, as economics. Hence it is not unlikely that these two methodologies get closer to each other, and create a common conceptual framework to analyze the dynamics of science in general.

It is not possible to propose such a framework here; this would require separate extensive studies. Instead, I propose a thought experiment. It should give just a first glance at Turing's (1939) idea of oracle as a mathematical model of inventive problem-solving.

Let us imagine that an ingenious engineer of strong AI produces a scientific robot (as defined in section 5) to simulate with UTM the historical Isaac Newton, to wit Newton's mechanical avatar, so to say. Let the proper name of that artefact be "T-Newton" to indicate its Turingian (1936) nature.

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[6] For instance, the article *Extension* by Marciszewski in: Dictionary ..., W. Marciszewski (ed.), 1981 (2013). *Available through books.google.pl after filling the search box with the term* Extension.

When programming T-Newton's brain, the designer must decide about the period of life in which acts such an artificial agent: should T-Newton be like young Newton, or more mature, or otherwise? Assume that the designer chooses Isaac Newton (1643–1727) in the age of twenty two, having the following properties: (1) already got to perfectly know mathematics, physics and astronomy as were available then to men of learning, but (2) he did not discover yet the universal law of gravitation. Shortly after, the twenty-three-year-old Newton made his famous legendary observation of falling apple. Let's capture that moment.

In a flash of intellectual enlightenment young Newton understood that the same force of gravity that pulled the apple to Earth kept the moon in orbit. Would it be likely in the case of T-Newton?

In order to try a response to this question, the AI-constructor would have endowed T-Newton's memory with identical content as that possessed by the real, twenty three years old, Newton. There must have been arithmetics, geometry and algebra, all of them highly in that time advanced, and besides the rules of logic necessary to prove theorems.

However, that is not all. Something more should be taken into consideration in order to appreciate the degree of Newton's inventive genius in comparison with T-Newton's abilities. A story to shed light at this issue is told below.

6.2.

The story should deliver a relevant example for debating on the inventive potential of Turing machine, personified in our tale by T-Newton. Let us assume that T-Newton's memory includes the *principle of impossibility of any action at a distance*. It says the following.

NAD: It is not possible for anybody to affect the other: (1) at any distance, (2) without requiring any portion of time, and (3) without any medium to carry the interaction.

The abbreviation NAD stands for the most concise Latin version: *Nulla Actio in Distans*. How obvious, certain and convincing seemed this principle to the most eminent thinkers, testifies the list of its adherents: Thomas Aquinas, Descartes, Leibniz, Broad, Michael Faraday, James Clerk Maxwell, Hendrik Lorentz, Heinrich Hertz, Albert Einstein.

Among them it was Leibniz who not only sticked firmly to NAD, but vehemently attacked and even ridiculed Newton's theory of gravitation for its giving up that inviolable and sacred principle. Leibniz's harsh satire bears the following title: *Antibarbarus Physicus pro Philosophia Reali contra* *renovationes qualitatum scholasticarum*.[7] This means. “Anti-barbaric Physicist in defense of realistic knowledge against the revival of occult qualities.”

English lexicons define “barbaric” as “marked by crudeness or lack of sophistication,” and this is what meant Leibniz when accused the Newtonian gravitation of being as crude, that is, lacking of sophistication, like naive explanation of Nature by the medieval schoolmen. In the Middle Ages this was a common expedient: properties lacking a known rational explanation—for example, magnetism—were considered occult qualities.

In the times close to those of Leibniz (1646–1716), it was Descartes (1596–1650) who claimed to eliminate occult qualities in favor of mechanistic explanation. This was exactly what also Leibniz defended as realistic knowledge (*philosophia realis*), and blamed the idea of gravitation as “chimerical.”

To see how much such a criticism was due to the *Zeitgeist* of the 17th century, let us notice its presence even in, so to say, “pop culture” of that time. It was Molier (1622–1673), comedy writer, who derided medician s of Sorbonne who biological phenomena, difficult for them to understand, treated in terms of occult forces: a scholastic doctor asked why opium makes one sleepy, replays: “for there is in it the force to make one sleepy”; instead of a scientific explanation—a linguistic trick.

While mechanism was endorsed then by progressive thinkers as the new paradigm to pave the way to the flourishing of science, Newton—now regarded the founder of mechanism in physics, seemed to betray that paradigm with his idea of gravitation. How to understand such a stance?[8]

Before answering this question, it is worth while to account for conception of mechanism worded by Leibniz. He devised the list of concepts of natural science which he regarded primary and fundamental, and apt to define remaining concepts of natural sciences. There he enumerated: number, measure, mass, shape, movement, and the relation of contiguity (maximal proximity) between bodies.

Significant is Leibniz’s claim that whatever happens in the physical universe, should be made conceivable in terms of contiguity and movement. This is why he could not believe in the reality of the gravitational force as moving bodies without their being in the relation of contiguity.

Leibniz did not deny a physical reality to such forces as those of magnetism, elasticity, etc. However he denied their being primary, i.e. fundamental. Instead, he allowed to use them as derived concepts, defined in terms of such primitive ones as movements and shapes—the sources of those secondary phenomena. His crucial statement is: “Permissum est agnoscere vires magneticas, elasticas aliaque sed ea lege ut intelligamus eas

[7] In: *Die Philosophische Schriften von Gottfried Wilhelm Leibniz,* von E. J. Gerhardt (ed.), vol. VII, Georg Olms, Hildesheim 1961, pp. 337–343, passim.

[8] As for Newton mechanism, see (Schiemann 2008, 36–38).

non esse primitivas sed ex motibus figurisque ortas" (see in: Gerhard 1961, 338).

This was a kind of reductionism which Leibniz agreed to apply to gravitation. In the case of such reduction, there was no need to assume that a body affects another one (1) at any distance (2) without requiring any portion of time, and (3) without requiring any medium to carry the interaction—as assumed by Newton.[9]

6.3.

Let me repeat the question taken as the title of the present section: *Would there be any dynamics of science, if the science were being done by machines?* The answer should be in the negative: if the science were being done by machines, then there would be no dynamics.

> It should be so, provided two assumptions: (A) "machine" means the universal Turing machine without oracle; (B) the dynamics of science does not consist in deriving new consequences from the axioms already existing, but in the inventing new axioms—such that some problems not being solvable on the basis of the former axioms become solvable after adding new ones.

The law of gravitation is like a new axiom added to the existing body of knowledge. Newton decided to do so in spite of seeing arguments for NAD. It is an interesting question whether T-Newton, programmed by his designer, would be able, after a reflection, to make such a choice—between pragmatism and fundamentalism—on his own. Fortunately for the future of science, he proved to prefer the pragmatist option.

We cannot learn his motivation, but leaving apart any psychological consideration, and judging just from a methodological point of view, we should appreciate his choice for the high level of *corroboration* characterizing his theory. I take the term "corroboration" in the sense defined by Karl Popper in his *opus magnum—The Logic of Scientific Discovery* (1959, chap. X).

However, Leibniz's belief in NAD has been supported by the most recent results in physics. Item 2 of NAD (see text box in 6.2) is to the effect that any physical interaction requires a portion of time was predicted by the Einsteinian theory of gravitation, but up to recently it remained beyond any experimental support. Only in 2015 the Laser Interferometer Gravitational-

[9] Compare the wording of NAD in the text box at the beginning of this section (6.2).

wave Observatory (LIGO) has detected gravitational waves which, according to Einstein's theory of gravitation are ripples in space and time. Hence gravitation proves to be a spatio-temporal phenomenon, as postulated by Leibniz; the speed of gravitational waves equals that of light.[10]

6.4.

The opinion that T-Newton, for the lack of invention, would not be able to make discoveries, and so contribute to the dynamics of science, might be objected with the following argument. The recent progress in programming makes it possible to build up systems with the capacity for adaptation, provided with mechanisms to allow them to decide what to do according to their objectives. Would it be enough to endow T-Newton with such a capacity that he effectively simulates the brain of historical Newton in all phases of his development?

Such T-Newton would then belong to the category called *autonomous agents*. They can react to events in their environment, to take the initiative according to their goals, to interact with other agents, to learn from past experiences to achieve current goals, to have propositional attitudes (belief, intention, desire etc.).[11]

The crucial question is to the effect: does a list like that above include agent's capacity to act against algorithmic instructions present in his program? Is it possible that the capacity to disobey the implemented program be acquired through self-programming, that is, a kind of learning? These questions arise from what we know about Newton's hesitations about NAD. Somehow he shared Leibniz's belief in the validity of that principle.

We know from biographical sources that Leibniz's intuition was not foreign to Newton. He had no reason to give it up before discovering the law of gravitation which refers to space (distance) but does not involve any mention of time—as demanded by NAD when rewritten as the following rule of research.

> R-NAD: *Do not attach to a system of assertions any sentence that does not meet the conditions 1-3 listed in NAD.*

R-NAD can be easily applied by a machine in a syntactic manner characteristic of algorithmic instructions. For example, the sentence "the gravitational force does not need any portion of time in order to affect a body"—contradicting item 2 in NAD—should be prohibited in any physical theory

[10] To learn more on this subject, see the page "LIGO detects gravitational waves for third time" by Massachusetts Institute of Technology.

[11] See J. M. Corchado et al. (eds.), *Web Engineering: International Conference, 2003. Proceedings*, Springer Science & Business Media, 2003, Chap. "Agent-Based Web Engineering."

(contradiction is a syntactic relation ascertainable with comparing strings of symbols).

Assume that the brain of historical Newton is a Turing machine (without any oracle!), as claimed in the strong AI approach. Assume that R-NAD is an algorithmic instruction implemented by a programmer to steer this machine's (Newton's brain) performances. *Is it possible for a Turing machine not to obey R-NAD?* It is the Key Question—KQ for short—in our case study.

6.5.

When taking for granted the assumptions stated above, together with the instruction R-NAD (in the text box in, §6.4), KQ should be answered in the negative. Such a disobedience is not likely to happen in the world of machines.

What, in fact, did happen to real Newton? As we can learn from his intellectual biography, he did not reject either NAD or R-NAD. So what happened? The closest to truth is the answer that he *suspended* his stance toward NAD. Would this be possible for T-Newton? Obviously not; this is forbidden to a machine to deny the validity of an algorithmic instruction. But what about the state of suspension?

This is a question to be addressed to experts in software engineering. However, from a philosophical point of view, in the domain of human relations such a suspension would be an act of disobedience. Imagine a clergyman in a religious community who does not reject its dogmas, but neither rejects them nor affirms them as valid, and remains in the mental state of suspension, or hesitation. This implies that he is not true to his obligations as a clergyman.

Let us regard this situation as analogical to that in which a machine stops to fulfill a specific instruction of its program, remaining able to realize the other ones; and—assuming furthermore—this disobedience proves much advantageous for handling problems to be solved. Should we (A) still remain convinced that we deal with a machine, or rather should we (B) come to the conclusion that the system in question is a certain non-mechanical entity?

It is B, the latter option, that is being argued for in the present essay, on the basis of the following historical evidence.

As mentioned above in 6.4, that objections like those of Leibniz were considered by Newton too. Before formulating his law of universal gravitation, he shared NAD as conviction, common to empirical scientists and philosophers. After attaining at the idea of gravitation he did not reject NAD, but rather took the position of suspending it, i.e., not taking it into account. Let this option be called *strategy of neutrality*. Now KQ (as raised at the end of 6.4) can be rewrite as follows.

KQ*: *Is it possible for a Turing machine to adopt the strategy of indifference with respect to R-NAD written into the machine's program?*

Such an indifference (neither accepting nor rejecting) was the policy of historical Newton with respect to the idea of gravitation. Could it be adopted by the artificial T-Newton? To solve this problem, one has to find Newton's motivation, and then think whether such a motivation be attainable for T-Newton?

Newton's motive can be summed up with the phrase: *success of corroboration*. The notion of cognitive success as considered in epistemology goes back to Peirce's pragmatism, enhanced by Kazimierz Ajdukiewicz (p. 9 ff). It is fittingly employed in William Harper's 1998 essay *Isaac Newton on Empirical Success and Scientific Method*. The kind of the success in question is named here *corroboration*, following Popper (1959, chap. X) who as first introduced this notion to the methodology of empirical sciences.

Everybody is familiar with the Popperian concept of falsification: an attempt at falsifying a hypothesis is a search for counterexamples, while its failure increases the degree of corroboration of that hypothesis. On the other hand, if falsification does succeed, this means the lack of corroboration.

However, the fact that a theory has withstood all rigorous tests for a long period of time does not mean a definitive corroboration. It just evidences that so far the theory of question has received such a degree of corroboration that it can be retained as hitherto the best available theory. Yet this does not imply its being safeguarded against a possible refutation in a future.

The more general is a hypothesis *h* and the more precise (i.e., free of vagueness) are concepts involved in it, the more it has empirically testable consequences, that is, ones exposed to the risk of proving false. If a consequence proves false, this falsifies its premise *h*, according to the logical law of transposition:

$((h \Rightarrow e) \perp \neg e) \Rightarrow \neg h$

where *h* represents the hypothesis under test, and *e* reports an experience intended to test *h*.

Newton's gravity hypothesis turned out to have an enormous reach. It gave birth to an immense set of consequences in the scale of the whole universe—something impossible to imagine from the beginnings of science up to the 17th century.

This meant an enormously high degree of falsifiability, and thereby testability. For, in such a multitude of empirical consequences, each of them is exposed to the risk of getting falsified with experiments, or other empirical observations performing the task of honest severe tests. Since in all the applied tests the gravitation hypothesis proves its mettle, it ought to be recognized as having an exceptionally high degree of corroboration.

Thus it becomes evident how to understand Harper's [1998] phrase "Newton's empirical success." It means an enormously high degree of corroboration. Newton himself so accounts his achievement (quoted after Harper 1998).

> "*It is enough that gravity does really exist and acts according to the laws I have explained, and that it abundantly serves to account for all the motions of celestial bodies*"

The phrase "it is enough" seems to indicate that he did not see any need to justify his attitude of indifference with respect to NAD. This pragmatic attitude neutralized fundamentalist scruples with which he "privately" (so to say) might have felt. Nevertheless finally he has been certain that what really matters it is the power of his theory to account for all the motions of bodies.

6.6.

Newton's coming to such a success can be explained in the terms of *computability* and *pragmatism*—listed in the title of this essay as main factors to drive the dynamic progress of science. Pragmatism may be briefly rendered with the Chinese proverb: "*Black cat or white cat: if it can catch mice, it's a good cat.*" The effect of catching is by Newton featured with the assessment that the theory of gravitation "abundantly serves to account for all the motions of celestial bodies."

The Newton case features how astonishing may be efficiency of pragmatist strategy to drive the progress of science. Imagine that Newton would have decided to scrupulously conform to NAD. Then he should have abandoned creating and publishing the theory of gravitation as incompatible with the principle whose inviolability was unanimously in that time acknowledged. Such a capitulation would have halted the splendid development of physics in which without Newton would have been no Einstein. This historical lesson reveals the enormous advantage of pragmatic strategy over the policy of fundamentalism.

However, with a paradoxical turn of history there happened that the principle NAD revived with Einstein's general relativity as dealing with the cosmological scale of magnitude. While in our mundane scale the light wave does not seem to need any time to travel any distance, it may take millions of years at interstellar distances.

The same applies to gravitational waves. Their existence was inferred by Einstein a hundred years ago from the equations of his theory of relativity, while experimental confirmation of this prediction occurred for the first time in 2015. The speed of this undulation proves to be like that of light waves. This discovery confirms item 2 in the NAD statement (text box in section 6.2), and possibly item 3 (id depends on how we understand "medium"), and so it resembles Leibniz's stance.

Einstein's theory would not have been arisen if not preceded by that of Newton. Thus in the 21st century we would have neither Newtonian nor Einsteinian understanding of the universe, both being a fruit of Newton's pragmatism.

In turn, let us consider the computational rationalism from the nowadays perspective and compare it with Newton's stance. Newton was a rationalist at least in that restricted sense that in his research he did not endorse such a radical empiricism as that suggestively articulated in the 20th century with the Vienna Circle. His practice displays a tint of Popper's critical rationalism since it is far from applying any logic of induction as postulated by the 20th century empiricists, esp. Hans Reichenbach.

Instead, Newton practiced something like the Popperian hypothetico-deductive realism. He does not try to justify a hypothesis by inferring it as a logical consequence from sensory observations (as demanded by inductive logic). He uses just deductive logic to derive observational consequences; if they do not disprove the hypothesis in question, it becomes to some degree corroborated.

Where do the hypotheses come from if they are not inferred from observational statements? The answer is brief: they arise from scientific inventiveness. It may consist in an intellectual intuition, as claimed by Gödel, or in a play of imagination, as stressed by Einstein. What does matter, is not their mental origin, but testability and the increase of corroboration after successively passing appropriate tests.

Thus we come to a point related to the title of the present section: would there be any progress of science, if the science were being done by machines? The answer might be in the affirmative, if the dynamics of science were due to the inductive strategy. T-Newton, as a specimen of Turing machine not being supported by an oracle, would obtain observations from, say, a camera. Then it would transform the obtained pictures into observational reports, and from them conclude a hypothesis using an algorithm of inductive logic.

However, one has to take into account that a logic of induction such as would be needed by T-Newton, serving the purpose of establishing universal laws of the universe, did not arise yet. Just a creative invention, able to produce testable hypotheses is the key factor to drive the dynamics of science. And the attribute of invention remains so far a privilege of humans alone, not of machines.

In turn, it is in order to combine the study of Newton's case, paradigmatic for empirical science, with what has been formerly (sections 1–4) said about dynamics of mathematical inquiries. Thus we recognize the same key factor characteristic of scientific dynamism in both areas. This decisive factor is represented by hypercomputational model defined by Turing (1939), to wit ORACLE—defined in section 3.1., the text box and then the indented quotation.

Turing is here dense in content but sparing in words. Anyway it is for him beyond any doubt that the oracle: (1) cannot be a machine; (2) with its help we can form a new kind of machine (*o*-machine), having as one of its fundamental processes that of solving a given number theoretic problems; (3) its answers are mostly, though not always, reliable solutions.

The so defined oracle is the key factor of the scientific dynamics. It is what overcomes the hitherto existing limits of knowledge, pushing its frontiers further and further. In mathematics this model gets realized with inventing new concepts and new axioms, while in empirical sciences—inventing new concepts and new hypotheses. Such a perspective on the evolution of science is worthy to be called computational and pragmatic.

Computational—in a broader sense. Not in the sense of reducing the model of science to the universal Turing machine (UTM) as a canonical paradigm of research procedures. This approach consists in dividing the class of problem-solving processes into those for which simple UTM is a sufficient device, and those which require an adequate sequence of oracle-machines. Both categories involve in their definitions the notion of computability.

If we look at the world as an immense system of problem-solving processes, for instance in the vein of A. N. Whitehead's processualism, and assume the above computation oriented classification of processes, then such a vision deserves to be called *computational worldview*.

The justification of the term "pragmatic" is given above to characterize Newton's approach, with commenting the maxim: "black cat or white cat: if it can catch mice, it's a good cat." Newton's case makes us aware that the pragmatic approach, even at the cost of disregarding what seems a fundamental intuition, may be awarded with obtaining a highly informative and highly corroborated theory.

Let me conclude with a tale about the fulfillment of the dream of strong AI adherents. They dream about the mechanical T-Newton that would be indistinguishable from the living Newton, as to the scientific achievements, and as to the way he came to them. In order to perfectly simulate real Newton, his mechanical avatar T-Newton should have had the following history.

A) Besides all the mathematical, physical and astronomical knowledge possessed by Newton in the age of 23, T-Newton should have believed in NAD and should have had in his memory the instruction R-NAD within the program implemented by the designer.

B) After having seen the legendary falling apple, he should have experienced a flash of enlightenment that there does exist the force of gravitation which acts—in any distance and without needing any time and any medium—between arbitrary bodies in the universe.

C) Having been aware of the inconsistency between R-NAD and this newly invented idea, he should have suspended the former, and so acted against the intention of his programmer; otherwise he would not have become the discoverer of gravitation.

Now any adherent of the strong AI project has to decide whether such an inobedience does accord with the definition of purely automatic (i.e., without oracles) Turing machine. He is not obliged to project T-Newton according to UTM model, but then he ought to reveal what other kind of machine would satisfy his intention: in such a way that an artificial Newton discovers the force of gravitation.

7. A FURTHER OUTLOOK: THREE MODELS OF THE DYNAMICS OF SCIENCE

7.1.

"Curiosity is more important than knowledge." — said Albert Einstein. More important for what? The answer proposed in this essay reads: for the progressive and accelerating dynamics of scientific knowledge.

Why more important than knowledge? Human curiosity brings about inventiveness—*the main source of the accelerating dynamics of science.*

Non-human animals have a knowledge, partly instinctive, partly acquired with experience, necessary to survive and to satisfy some biological needs. However, with chimps, cats, dogs, etc., such a *knowledge* does not result in the *curiosity*, and the elicited with it *inventiveness*, while from such attributes alone there could be born Platon's philosophy, Euclid's geometry, Copernicus' astronomy, Frege's logic.

Such a singularity resulted in the surprising dynamics of science. This phenomenon has become the subject of an intense study in the past century, and has forerunners already in the 19th century, but a flamboyant development started up in sixties of the 20th century in two separate directions. One of them has been initiated by de Solla Price (1963) the other one—by Thomas Kuhn (1962). There were quite a number of other penetrative studies on modelling the progress of science, carefully analyzed by Paweł Polak in his 2004 book entitled *Dynamika nauki* [Dynamics of Science].

That book influenced the present essay considerably by supplying a clear and comprehensive overview of scientific dynamics models. I take an essential advantage of this survey. It perfectly serves as a contrastive background which makes it easier to point to one more model. This one has not been discussed hitherto under the title "dynamics of science," for its having been classified under the label of the *logico-mathematical theory of computability*. However, for anyone who happens to be interested both in the issues of computability and in the historical development of science, the nexus between them is evident. The present author's intention is to make this evident for a wider audience as well.

Professor Polak discusses no less than fifteen models, but for the present comparative task it will suffice to consider just two of them—those mentioned above, one due to de Solla Price, the other to Kuhn. Such a selection is justified by their pioneering role (see above) and by the fact that they enjoy incomparably more attention and influence than the remaining approaches.

The following abbreviations should make the discourse shorter and more transparent:

exp-model — de Solla Price's *exponential* model.

par-model — Kuhn's *paradigm* model.

orc-model — Turing's *oracle* model.

They are to be defined, and compared with each other, in the next subsections.

7.2. Models of the dynamics of sciences: exp-model compared orc-model

The exponential model is applied to characterize the speed of growth of a population, that is, the set of individuals, items, or data from which a statistical sample can be taken. When speaking of science, we have to do with the sets of scientists, publications, theories, academic journals and institutions. It was de Solla Price (1963) who considered the dynamics of such populations from the half of the 17th century up to the half of 20th century, and for that period has found instructive generalizations.

In each of these populations the dramatic exponential growth occurred. To wit, systems that exhibit exponential growth have a constant doubling time. The quantity increases slowly at first, and then very rapidly. The rate of growth becomes faster as time passes. Thus the size of science measured with number of scientists, or number of publications, doubles every 10 to 15 years. As a result, science has been constantly exploding, increasing its size at a rate faster than the increase of total humans able to conduct it.

However, de Solla Price assumes that the exponential growth rate may be starting to diminish. This is to mean that that the growth may proceed until it reaches a maximum size and then ceasing to grow. If science had continued to grow at an exponential rate in 1962, then by now there would be more scientists than people. Thus the exponential growth rate previously observed must break down at a point in the future, and this breakdown may be associated with an upper bound to the size of science reached in the period of the flamboyant expansion.

This de Solla Price's exp-model is the subject of a sophisticated statistical theory, having plenty of fertile applications in sociology, information sciences, politics, etc. That fact should not be obscured by the paucity of the account given above. This account is restricted to what is indispensable to compare exp-model with Turing's orc-model in a most crucial point.

It is the point of a further outlook. In orc-model it is shaped with Turing's (1939) idea of *oracle*. In the present text, this idea is found at the very front of the key concepts' list (as having ca. 30 occurrences in the present text). In the section 3.1 there are mentioned the authoritative Turing's utterances concerning the nature and role of this device in problem-solving processes (see the text box, and then the paragraph distinguished with indenting).

There are two facets of comparison between exp-model and orc-model: how to understand and measure the size of science, and how to predict its future development. In either respect these models are complementary. The exp-model is concerned with what can be termed *external size* while orc-model—internal size. The former is measured through various quantities, as listed above; let us focus on the quantity of publications in a given point of time (let it be the year of publication).

The latter is to be seen as the function of two variables: (1) how big is the increase of the number of problems having been solved by a new publication, (2) how much difficult, and how much important, are the offered solutions (this is a very sketchy featuring, but it will do in the present discussion).

Let us look at some most dramatic instances, those of Newton, Einstein and Frege. Newton's *Principia* appeared in 1687. The external size of the physical science increased thereby by one item only; not more than in the case of, say, a typical doctoral dissertations published in the same year. As for the increase of internal size, Newton's work should be assessed with the possibly highest index. The same ought to be said about Einstein's 1915 work on general relativity (the geometric theory of gravitation), and in field of logic about Frege's *Begriffsschrift* (1879). In each case the increase by one in the external size has resulted in the immense increase of the internal size.

Correspondingly, we should distinguish between the *external dynamics* of science that consists in a considerable increase of external size, and the science's *internal dynamics*—considerable increase of internal size.

Such a striking difference between these two notions concerning the size of science sheds light on the issue of their further outlook. When the growth of external size has to slow or even stop for exhausting necessary physical resources, such limitations do not appear in the case of internal size when considered in terms of orc-model; theoretically there is thinkable an unending progress into infinity. To explain such a disparity, we have to inspect a bit deeper into the nature of orc-model.

7.3. Models of the dynamics of sciences: par-model compared with orc-model

Kuhn's par-model (1962) has won a wide acceptance in academic circles. In order to estimate the mettle of his theory, we should consider it separately for mathematics and for empirical sciences. As for the former, par-model is directly contradicted by orc-model which does not envisage any *scientific revolution*—for Kuhn the basic notion in considering *paradigm shifts*.

The latter notion denotes—with Kuhn—a fundamental change in the basic concepts and practices of a scientific discipline. The paradigm shift characterizes scientific revolution, contrasted with the activity of normal science, that is, scientific work done within a prevailing framework or paradigm. Paradigm shifts arise when the dominant paradigm under which normal science operates is rendered incompatible with new phenomena, facilitating the adoption of a new paradigm.

Let us try to adopt this characterization to what we know about the history of mathematics. As the first and absolutely dominant at any time paradigm we should acknowledge Euclid's *Elements*. It is a perfect opportunity to put some questions which may prove troublesome for Kuhn's adherents.

Did a shift of this paradigm happen at any time? If one tried to imagine such an event, at most two facts in the history could be considered. (1) This might be modern axiomatizations: of geometry, like that by Hilbert, and of arithmetic, like that by Peano. (2) It might be as well the rise of non-Euclidean geometries. Let us suppose that these are questions open to discussion. As its participant, in both cases I would answer in the negative.

— Hilbert did not oppose Euclid's paradigm of axiomatization, but just perfected it with the help of modern logic. Should this be regarded as a scientific revolution?

— Non-Euclidean geometries (e.g., hyperbolic) can be interpreted within Euclidean geometry. Should this be regarded as a scientific revolution?

— Peano's axiomatization did not change any theorem of arithmetic, it just added a new method of proving theorems. Should this be regarded as a scientific revolution?

— Frege's mathematical logic evolved—via Leibniz's design of logic, and Boolean algebra—from the traditional Aristotelian logic which has been then absorbed by modern logic as its small chapter. Should this be regarded as a scientific revolution?

I cannot answer in the affirmative to any of the above and similar questions. If anybody can, her/his voice will be welcome. Kuhn's idea of scientific revolutions was intended itself as a revolutionary manifest against the cumulative vision of science. However, in confrontation with historical facts, the latter proves its mettle, while the revolutionary appears as utopian. Cumulativeness means that upper storeys of science are put over the lower and lower ones, up to foundations.

Raine and Heller (1981) rightly notice that in the historical development a newer theory happens to involve an older one on the pattern of a more advanced generalization. This was the case of Einstein's theory of gravitation which encompassed the Newtonian as a special case—a useful approximation in certain conditions. Is there any reason to regard such a transformation as a revolutionary overturn? Let us note that such a thrilling approach (as in the sensational press) makes us unable to trace evolutionary processes in order to understand mechanisms which cause the passing from one stage to another.

However, the appreciation of the rule of cumulativeness (as with Raine and Heller (1981)) does not suffice to grasp the explanatory power of Turing's orc-model. We need a bit more detailed insight into the evolutionary pattern of science dynamics.

There is the English term "mavericity" coined from the name of Maverick, a legendary Texas pioneer who applied very unusual methods in breeding his cattle. Hence *mavericity* denotes a quality *to generate unusual, uncommon, interconnections between ideas* or to do something unexpected (cp. Runge 2014, 280).

The highest internal dynamics of science (cp. Section 7.2)—that exemplified with Newton's and Einstein's gravitation, quantum theory, Frege's logic, Gödel's incompleteness theorem, Turing machine—does result from an encounter of new ideas is marked with a high degree of mavericity.

Newton's theory of gravitation stems from the encounter of (1) Kepler's planetary model, (2) invented by Newton Calculus, (3) Newton's flash of understanding the nexus between falling fruits and planetary movements to satisfy Kepler's model.

Einstein's theory of gravitation would not arise without the encounter of his principle of equivalence (of gravitational and inertial mass) and Riemannian elliptic geometry.

7.4. On how the computer science has emerged from the encounter of several surprisingly inventive ideas

When availing myself with the term "inventive" as a highly significant key notion in the present context, I was wondering what adjective might render the highest possible degree of this trait: exceptional, uncommon, unusual? "Surprising" seemed the best. Namely, for its connection with the theory of probability and information in which "surprise" happens to be used to idiomatically hint at a great novelty of message, that is, a great amount of information.

In slangish American, as mentioned above, there is a word close in its meaning to "surprising inventiveness." It is: *mavericity* (cp. 7.3). Such a single word is more handy in a frequent use, hence I allow myself to use it as a convenient abbreviation.

Mavericity is a specific feature of creative humans, the ability to do something unexpected, by no means attainable for any kind of automata. When an agent or device has been programmed, i.e. acting according to an algorithm, it cannot do anything unexpected. When can, it is not mechanical, and this is exactly what Turing says about any oracle—as being an agent capable of solving mavericitly some problems whose solution cannot be expected from any machine.

Turing's featuring of oracle is done in abstract mathematical terms. As mentioned above (3.1, footnote 2) there is a convincing interpretation of oracle in a mentalistic way—a mathematical intuition. However, the sharing of that interpretation does not exclude proposing still another one. What I am to suggest deserves to be called a historical interpretation of oracle, and should provide an oracle-oriented model of scientific dynamics.

Historical mavericity appears when (2) there is a number of ideas, each having been developed without any connection with other ones, and (3) there comes the moment when they encounter dynamically, combine with other and affect mutual gravitation. From such tectonic events happen to emerge new unexpected whole. The result of such an emergence is worth to be termed *oracle* in a historical sense—as much as it brings significant solutions of some problems, having been had no chance of solution without such a dramatic encounter.

As historical examples it has been mentioned in 7.3 the encounters of Newton's and Einstein's theories of gravitation with needed by them geometries. As a more complex case of encounter in which quite a number of participants appears, we can take the long process leading to the emergence of computer science. The process has the physical and the logico-mathematical side. Leaving apart the former, in the latter we notice at least seven stories. Their list would include the following items:

Aristotelian logic
Leibniz's project of conceptual writing
Algebraic ideas: Viete, Peacock, Boole
Frege: conceptual writing, axiomatized logic
Cantor's diagonal method of proof
Gödel's arithmetic coding of the language of logic
Universal Turing Machine

A story about the processes of connecting and mutual support of these ideas would take an extensive treatise. Let it be only noted that it was Frege who combined Leibniz's ideas of a perfect "*characteristica universalis*" and "calculus ratiocinator," with basing his logical calculus on the Boolean algebra which, in turn, emerged from combining Aristotelian logic with algebraic achievements, going back to François Viète, George Peacock etc. The construction of Turing machine involved Cantor's method of diagonal proof and Gödel's coding.

Even such a drastically sketchy account allows to see that no single item at the above list, if taken apart, could solve the problem of "how to construct computer," while their sophisticated combination with each other and with relevant idea of physics resulted in so potent computing machines.

Thus we obtain an exemplification of the powerful and unstoppable dynamics of science, reflected by the model of oracle, and being due to the power of surprising inventiveness (mavericity) of human beings.

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